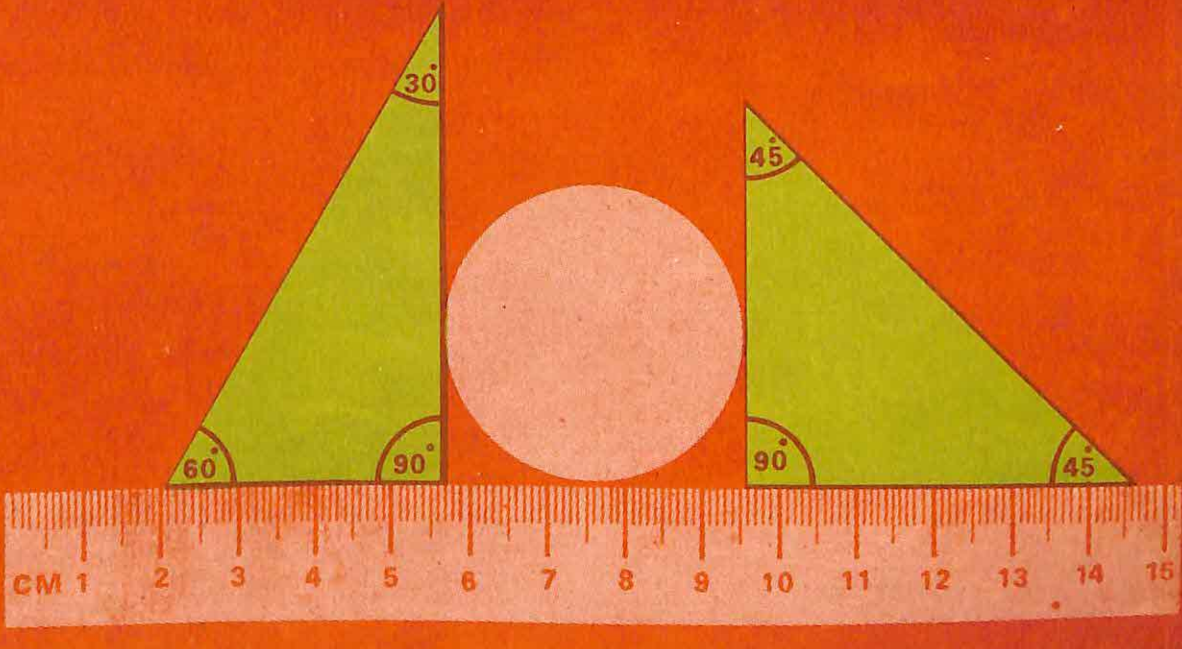
A NEW STYLE
GEOMETRY

BOOK VIII

TEJ RAM & PITAMBAR LAL



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A
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GEOMETRY

BOOK VIII

By

TEJ RAM, *M.A.*

And

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PREFACE TO THE FIRST EDITION

We take great pleasure in presenting to the young students this series of books on Geometry. The dearth of suitable books in the market with Indian background encouraged us to write the present series. In preparing this series our chief aim has been to present the principles of the subject in a most easy-to-understand and easy-to-work out manner. The subject matter has been presented in a very lucid manner and the language is easy.

Throughout the book the sound principle of taking difficult questions one at a time has been followed. Typical examples of almost every variety have been solved and in the ensuing exercises questions based on these examples have been set. Particular care has been taken in the classification and gradation of the subject matter. Clear diagrams have been supplied wherever necessary to make the reading instructive as well as interesting.

The answers given at the end of the book have been thoroughly checked.

It is hoped that these books will meet the needs of the students for whom they are meant.

– Authors

PREFACE TO THE LATEST EDITION

In view of the ever increasing demand of this book, we have undertaken to bring this book in new size and shape.

All the diagrams have been drawn anew. Some diagrams have been improved. Matter has been revised at a few places in accordance with the suggestions received.

We hope this new get-up and size will make the book easier to read and grasp.

Suggestions for improvement are welcome.

— Publishers

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CHAPTER I

SOME PROPERTIES OF TRIANGLES

1. We have already studied the following geometrical facts in the earlier classes :

(*a*) When a straight line meets another straight line at a point, the sum of the adjacent angles so formed is equal to two right angles.

(*b*) If the sum of two adjacent angles is equal to two right angles, their external arms are in one and the same straight line.

(*c*) If a straight line meets another straight line, the vertically opposite angles are equal.

(*d*) The sum of the three angles of a triangle is equal to two right angles.

(*e*) Four theorems on congruency of triangles :

(*i*) If two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the other, the two triangles are congruent.

(*ii*) If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of the other, the two triangles are congruent.

(*iii*) If the three sides of one triangle are respectively equal to the three sides of the other, the two triangles are congruent.

(*iv*) If the hypotenuse and a side of one right-angled triangle are respectively equal to the hypotenuse and a side of the other right-angled triangle, the two triangles are congruent.

(*f*) If two sides of a triangle are equal, the angles opposite to these sides are equal.

(*g*) If two angles of a triangle are equal, the sides opposite to these angles are equal.

(*h*) When a straight line cuts two parallel straight lines,

(*i*) the alternate angles are equal,

(*ii*) the corresponding angles are equal, and

(*iii*) the interior angles on the same side of the straight line are equal to two right angles.

(*k*) When a straight line cuts two other straight lines such that

(*i*) a pair of alternate angles is equal, or

(*ii*) a pair of corresponding angles is equal, or

(*iii*) a pair of interior angles on the same side of the cutting line are together equal to two right angles, the two lines are parallel.

The student may easily verify any of these theorems by drawing suitable figures.

These geometrical facts are called **Theorems.**

A theorem consists of two parts.

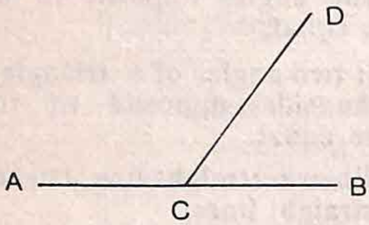
Consider the theorem :

If two straight lines intersect, the vertically opposite angles are equal.

(*i*) The **hypothesis** or that which is given or assumed, *e.g.*, "two straight lines intersect", and

(*ii*) The **conclusion** or that which is to be proved, *e.g.*, "the vertically opposite angles are equal".

Ex. Point out the hypothesis and the conclusion in the theorem :



If a straight line stands on another straight line, the sum of two adjacent angles so formed is equal to two right angles.

The hypothesis is "A straight line stands on another straight line", *i.e.*, CD stands on AB.

The conclusion is "The sum of the adjacent angles so formed is equal to two right angles", *i.e.*,

$$\angle ACD + \angle BCD = 2 \text{ rt. } \angle\text{s.}$$

If we change the hypothesis into a conclusion and the conclusion into the hypothesis, the new theorem becomes that is known as the **converse** of the original theorem.

Thus the converse of the above theorem will be

"If the sum of two adjacent angles is equal to two right angles, their external arms are in one and the same straight line."

Here we know that "the sum of two adjacent angles is equal to two right angles", *i.e.*, $\angle ACD + \angle BCD = 2$ rt. $\angle$s. (hypothesis) and we prove "their external arms lie in the same straight line" *i.e.*, ACB is a straight line (conclusion).

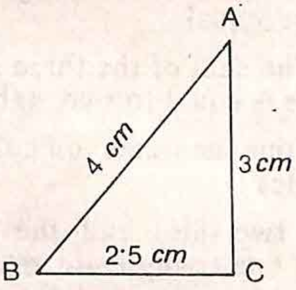
Note. It must, however, be clearly understood that the converse of a theorem is not always true. This will be evident from a simple illustration :

"If the animal is a cow, it must have horns" is true. But its *converse*

"If an animal has horns, it must be a cow" is evidently not true.

Now we shall study some more theorems.

Ex. 1. Draw a $\triangle ABC$ in which $AB = 4$ *cm*, $AC = 3$ *cm*, and $BC = 2{\cdot}5$ *cm*.



Here $AB > BC$.
Measure $\angle ABC$; it is 49°.
Measure $\angle ACB$; it is 93°.

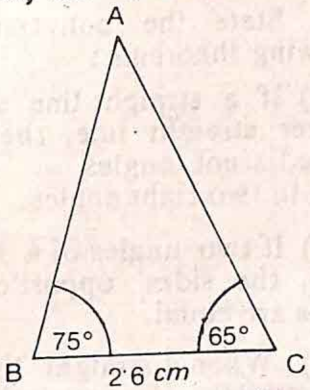
That is the angle ACB opposite to the greater side AB is greater than the angle ABC opposite to the smaller side AC.

Ex. 2. Repeat the experiment with different triangles ABC, always taking AB greater than AC. $\angle C$ will always be found greater than $\angle B$.

Hence we conclude :

If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.

Ex. 3. Draw a $\triangle ABC$, on base BC $= 2{\cdot}6$ *cm* long. At B make $\angle ABC = 75^\circ$, and at C, $\angle ACB = 65^\circ$. Let their arms meet in A. Measure AB and AC. $AB = 3{\cdot}5$ *cm*, $AC = 3{\cdot}8$ *cm*.



$\therefore$ AC, the side opposite to the greater angle is greater than AB, the side opposite to the smaller angle.

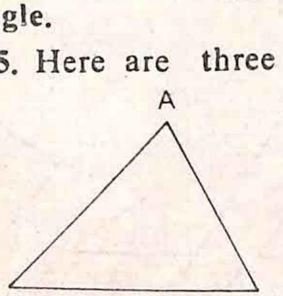
Ex. 4. Repeat the experiment by taking

(*i*) $BC = 19$ *cm*, $\angle ABC = 75^\circ$, $\angle ACB = 45^\circ$,
(*ii*) $BC = 3$ *cm*, $\angle ABC = 60^\circ$, $\angle ACB = 40^\circ$.

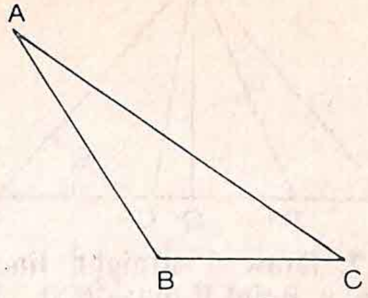
Check by actual measurement that AC is greater than AB. Hence we conclude that

If one angle of a triangle is greater than another, then the side opposite to the greater angle is greater than the side opposite to the lesser angle.

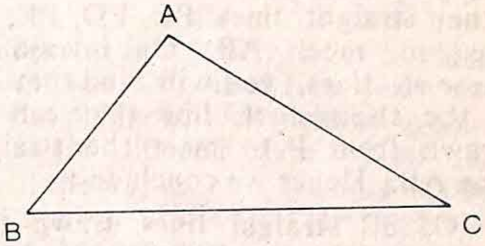
Ex. 5. Here are three triangles. Measure their sides and tabulate the results as shown below:



(i)



(ii)



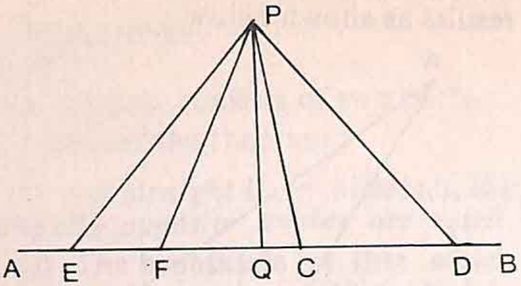
(iii)

Fig.	$b+c$	a	$c+a$	b	$a+b$	c	Conclusion
(*i*)							Any two sides are together greater than the third
(*ii*)							Do
(*iii*)							Do

Ex. 6. Repeat the same experiment with some other triangles. It will be found that any two sides of the triangle are together greater than the third.

We conclude that

Any two sides of a triangle are together greater than the third.



Ex. 7. Draw a straight line AB and take a point P outside it. Draw PQ perpendicular to AB. Also draw other straight lines PC, PD, PE, PF etc., to meet AB. On measuring these st. lines, you will find that PQ is the shortest st. line that can be drawn from P to meet the straight line AB. Hence we conclude :

Of all straight lines drawn to a given straight line from a given point outside it, the perpendicular is the shortest.

Exercise 1

1. Point out the hypothesis and the conclusion in the following theorems :

(*a*) If two straight lines intersect, the vertically opposite angles are equal.

(*b*) The sum of the three angles of a triangle is equal to two right angles.

(*c*) If the hypotenuse and a side of a right-angled triangle are respectively equal to the hypotenuse and side of another right-angled triangle, then the two triangles are congruent.

(*d*) If two sides of a triangle are equal, the angles opposite to these sides are equal.

(*e*) When a straight line cuts two other straight lines such that a pair of alternate angles is equal, the two st. lines are parallel ?

2. What do you mean by converse ? Is converse of every statement true ?

3. State the converse of the following theorems :

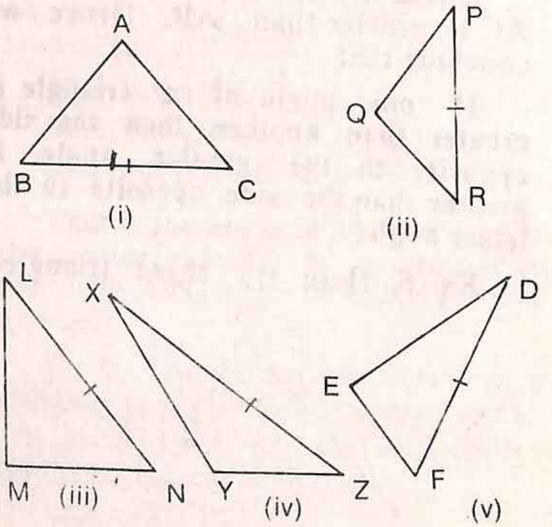
(*a*) If a straight line stands upon another straight line, the sum of the two adjacent angles so formed is equal to two right angles.

(*b*) If two angles of a triangle are equal, the sides opposite to these angles are equal.

(*c*) When a straight line cuts two other straight lines such that a pair of corresponding angles are equal, the two straight lines are parallel.

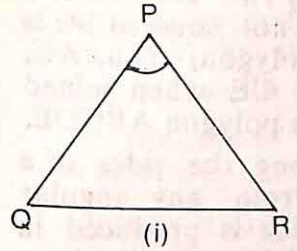
(*d*) If two sides of a triangle are unequal, the greater side has the greater angle opposite to it.

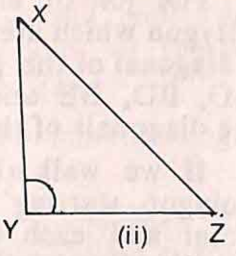
4. In the following triangles, the greatest side has been marked. Without measuring, tell which is the greatest angle in each case.

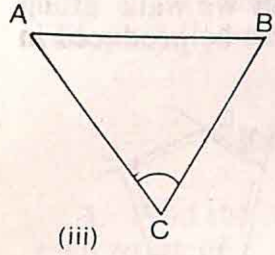


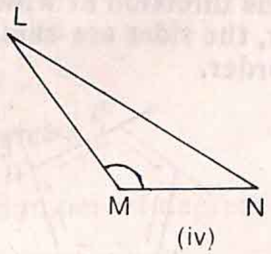
(i) (ii) (iii) (iv) (v)

5. In the following triangles, the greatest angle has been marked. Without measuring point out the greatest side in each case.







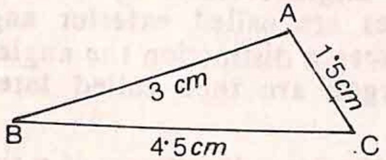


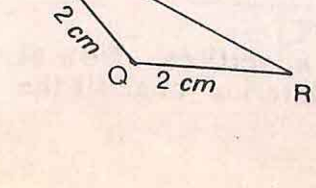
6. In a triangle ABC, ∠A=92°. Which is the greatest side ?

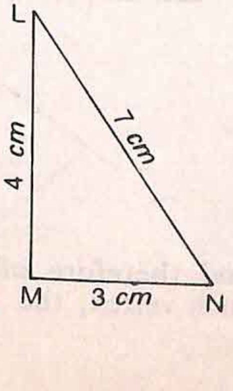
7. In a triangle ABC, ∠C=75°, ∠B=60°. Which is the greatest side ?

8. In a triangle $a=5\ cm, b=4\ cm, c=2\ cm$. Which is the greatest angle ?

9 State if the following triangles with the lengths of their sides marked on them are possible. If not, why not ?







Ex. 8

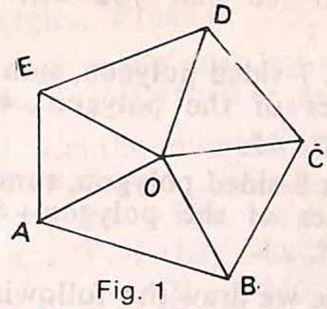


Fig. 1

Figure 1 is a polygon of 5 sides. The vertices are joined to a point inside the polygon. How many ∆s are seen in the figure ? Five. Now the *sum* of the angles of a triangle is two right angles. Hence *all* the angles of *all* the ∆s together

$$=5\times2 \text{ rt. } \angle\text{s.}$$

But the sum of the angles at O is equal to 4 right angles.

And all the angles of all the ∆s make up all the *angles* of the figure together with the angles at O, which are equal to 4 rt. ∠s.

∴ all the angles of the figure+4 rt. ∠s.

$$=5\times2 \text{ rt. } \angle\text{s.}$$

In figure 2, we have a polygon of

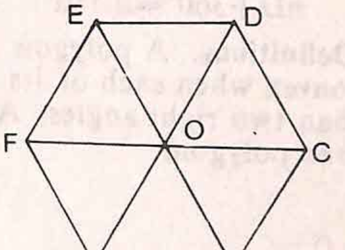


Fig. 2

6 sides, and we get 6 triangles. Hence sum of all the angles of all the ∆s =6×2 rt. ∠s. But the sum of the angles at O=4 rt. ∠s.

∴ all the ∠s of the figure+4 rt. ∠s=6×2 rt. ∠s.

Ex. 9. Draw polygons of 7 and 8 sides, and see that you will always find that :

For a 7-sided polygon, sum of all the angles of the polygon+4 rt. ∠s =7×2 rt. ∠s.

For a 8-sided polygon, sum of all the angles of the polygon+4 rt. ∠s =8×2 rt. ∠s.

Hence we draw the following conclusion :

The sum of all the angles of a polygon together with four right angles is equal to twice as many right angles as the polygon has sides.

Note 1. The above result can be put in this way :

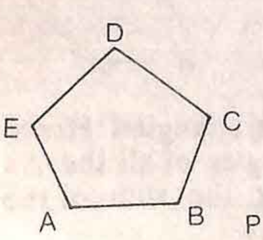
In a polygon of n sides, the sum of the angles is equal to $2n-4$ *right angles.*

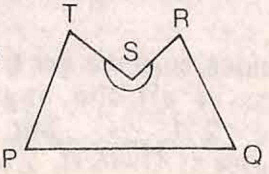
Note 2. A regular polygon is one which has all its sides equal and all its angles equal.

Thus if D denotes the number of degrees in each angle of the regular polygon of n sides, the above result may be stated thus :

$$nD+360^\circ=n.180^\circ$$

III **Definitions.** A polygon is said to be **Convex** when each of its angles is less than two right angles. ABCDE is a *convex* polygon.

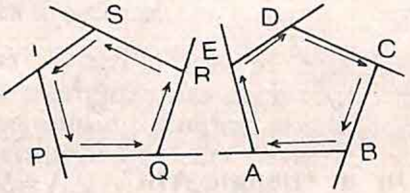




A polygon is said to be **Concave** when one or more of its angles are greater than two right angles. **PQRST** is a *Concave* polygon because ∠S is greater than two right angles.

The join of any two vertices of a polygon which are not consecutive is **a diagonal** of the polygon. Thus AC, AD, BD, BE and CE when joined are diagonals of the polygon ABCDE.

If we walk along the sides of a polygon starting from any angular point and each side is produced in the direction in which we walk along it, the sides are said to be produced in order.

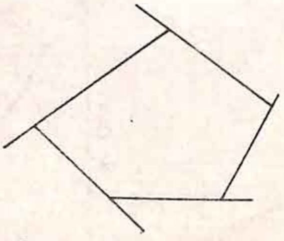


In PQRST each side is produced in a *Counter-clockwise direction* (*i.e.*, against the direction in which the hands of a clock move) and in ABCDE each side is produced in a *Clockwise direction.*

Note that no two sides when produced in order. intersect.

The angles formed by producing the sides are called **exterior angles,** and to keep a distinction the angles of the polygon are then called **interior angles**.

Ex. 10. Here is a figure of n sides,



and, therefore, of n vertices. Now at each vertex, the interior angles+the exterior angles=2 rt. $\angle$s ; and there are n vertices.

$\therefore$ The sum of the int. $\angle$s and the ext. $\angle s=2n$ rt. $\angle$s.

But we know (see the conclusion after Ex. 9) that the sum of the int. $\angle s+4$ rt. $\angle s=2n$ rt. $\angle$s.

$\therefore$ the sum of the exterior angles $=4$ right $\angle$s.

Note : It can be easily seen that these results are applicable to triangle and quadrilateral also.

Exercise 2

1. Find the number of degrees in each angle of

(*i*) a regular hexagon (6 sides)
(*ii*) a regular octagon (8 sides)
(*iii*) a regular decagon (10 sides).

2. In a quadrilateral ABCD, the angles B, C, D are respectively equal to 2A, 3A and 4A ; find all the angles in degrees.

3. Four angles of any pentagon (5 sides) are 40°, 78°, 122° and 135° ; find the fifth angle.

4. In any *regular* polygon of n sides, each angle contains $\frac{2(n-2)}{n}$ right angles. Prove it.

5. How many sides has a regular polygon each of whose angles is 108° ?

Sol. Let the number of sides be n. Then we have

$$n \times 108^\circ + 360^\circ = n.\ 108^\circ.$$

$$\therefore n.\ 180^\circ - n.\ 108^\circ = 360^\circ.$$

or $$72^\circ.\ n = 360^\circ. \quad \therefore n = 5.$$

6. How many sides has a regular polygon whose each angle is

(*i*) 120° (*ii*) 135° (*iii*) 144° ?

7. Is it possible to have regular polygon whose interior angles are :

(*i*) 180°, (*ii*) 120°, (*iii*) 36°, (*iv*) 130°, (*v*) 165° ?

8. Show that if one side of a regular hexagon is produced, the exterior angle is equal to the interior angle of an equilateral triangle.

9. Express in degrees the magnitude of each exterior angle of (*i*) a regular octagon (8 sides), (*ii*) a regular decagon (10 sides).

10. How many sides has a regular polygon if each of its exterior angle is (*i*) 30°, (*ii*) 24° ?

* * *

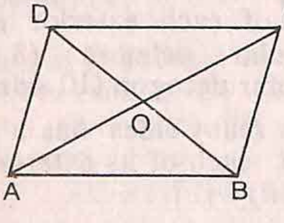
CHAPTER II

PROPERTIES OF PARALLELOGRAM, SQUARE, RECTANGLE AND RHOMBUS

We know that a **parallelogram** is a quadrilateral whose opposite sides are parallel. Now we shall study some of the properties of parallelograms.

Ex. 1. Draw a parallelogram whose adjacent sides are 3 *cm* and 2 *cm* long and included angle 75°. Join the diagonals as shown in the figure here, intersecting each other at O.

You may use the protractor and the set squares for this exercise.



(*i*) Measure AB and DC. They are equal, each = 3 *cm*.

(*ii*) Measure AD and BC. They are equal, each = 2 *cm*.

(*iii*) Measure angles A and C. They are equal, each = 75°.

(*iv*) Measure angles B and D. They are equal, each = 105°.

(*v*) Measure and see that $AO = OC$, $OB = OD$.

Ex. 2. Repeat this experiment with another parallelogram whose adjacent sides are, say, 5 *cm* and 7 *cm* long, and the included angle 60°. Note down by measurement the results (*i*) to (*iv*) of Ex. 1.

From these, we conclude that

(*i*) **the opposite sides of a parallelogram are equal ;**

(*ii*) **the opposite angles are also equal ; and**

(*iii*) **each diagonal bisects the other diagonal.**

Ex. 3. Draw any parallelogram ABCD, and draw its diagonal AC. Cut off carefully this figure along AB, BC, CD and DA with the help of scissors. Then cut this paper parallelogram along AC also. Try to fit the two pieces, one with the other. You will find that the $\triangle$s ADC and ABC coincide.

This shows that the diagonal bisects the parallelogram.

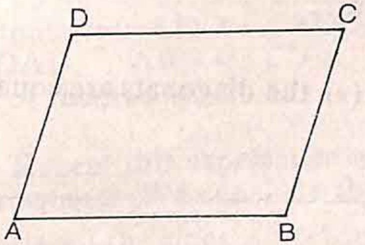
Ex. 4. Repeat the experiment by drawing another parallelogram ABCD and this time cut the paper parallelogram along the diagonal BD. The two $\triangle$s that we get again coincide. Hence BD also bisects the parallelogram.

Hence we conclude that

(*iv*) each diagonal of a parallelogram bisects it.

Ex. 5. Draw a quadrilateral ABCD, in which AB = DC = 3·5 *cm*, CB = DA = 2·5 *cm*. Note that the angle between the adjacent sides AB and AD can be taken of *any* size.

Now check by means of your set squares that AB is parallel to DC and AD is parallel to BC.



That is to say, the opposite sides of this quadrilateral are parallel.

Hence ABCD is a parallelogram.

Ex. 6. Draw two st. lines AD and BC parallel to each other and also equal. Join CD and BA. We get the figure ABCD. Measure AB and CD and check that they are parallel also. Hence ABCD is a parallelogram.

Ex. 7. Draw two intersecting straight lines AOC, BOD.

Cut off OD = OB, and OA = OC. Join AB, BC, CD, DA.

By means of your set squares see that AB is parallel to DC and AD is parallel to BC. Hence ABCD is a parallelogram.

Repeat the above three experiments by making different constructions.

We conclude that

A quadrilateral is a parallelogram if

(*i*) the opposite sides are equal, or

(*ii*) one pair of opposite sides are equal and parallel, or

(*iii*) the diagonals bisect each other.

Now we have verified the following theorem in Ex. 1 to 4 :

In a parallelogram

(*i*) the opposite sides are equal,

(*ii*) the opposite angles are equal,

(*iii*) the diagonals bisect each parallelogram, and

(*iv*) the diagonals bisect each other.

In Ex. 5 to 7 we have verified its *converse*.

A quadrilateral is a parallelogram, if

(*i*) the opposite sides are equal, or

(*ii*) one pair of opposite sides are equal and parallel,

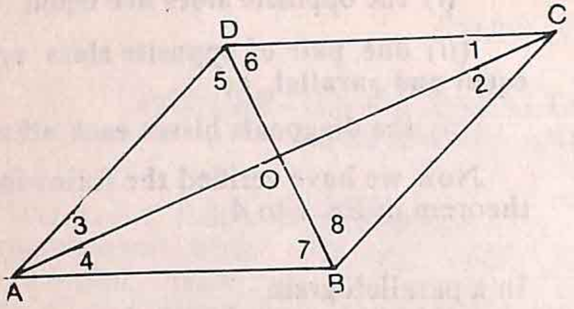
(*iii*) the diagonals bisect each other.

To this we may add another fact, which we have not verified.

(*iv*) A quadrilateral is a parallelogram, if the opposite angles are equal.

We shall prove it further on in another chapter.

Ex. 8. Draw a rhombus whose side is 4·2 *cm* and one angle 45°.



Verify by actual measurement that

(*i*) the opposite angles are equal,

(*ii*) the diagonals bisect the rhombus.

(*iii*) the diagonals bisect each other.

All these properties are common with a parallelogram.

Now measure ∠1 and ∠2. See that they are equal.

Also check that ∠3=∠4, ∠5=∠6, ∠7=∠8, *i.e*, the diagonals bisect the angles at the vertices....(*iv*).

Again, measure ∠s AOD, COD; ∠s AOB, COB. Each of them is a right angle.

Hence the diagonals of a rhombus bisect each other at right angles....(*v*).

These two properties (*iv*) and (*v*) are additional properties.

Repeat the Ex. with other rhombii (plural of rhombus) and check the above properties.

Ex. 9. Draw a rectangle with adjacent sides 5 *cm* and 3 *cm* long.

Check by actual measurement that

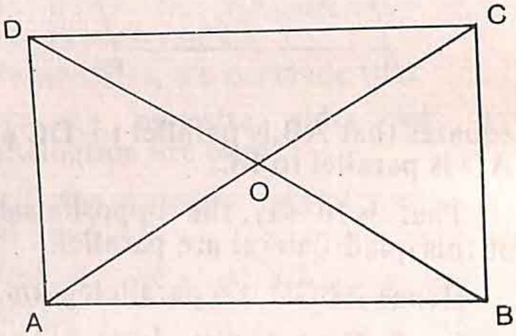
(*i*) the opposite sides are equal,

(*ii*) all the angles are right angles

(*iii*) the diagonals bisect the rectangle,

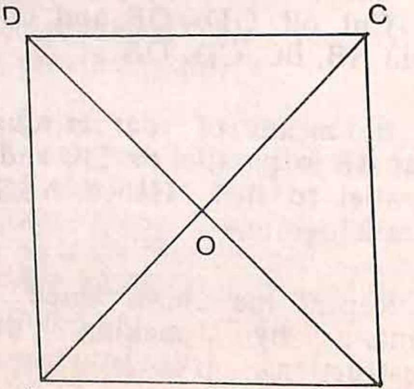
(*iv*) the diagonals bisect each other.

(*v*) the diagonals are equal.



The last property is not true for an ordinary parallelogram or even for a rhombus.

Ex. 10. Draw a square ABCD with side AB=4·5 *cm.*



Measure ∠s AOD, AOB, COB, COD. Each of them is 90°.

Measure AC and BD. They are equal.

Measure AO, OC; also OD, OB. They are all equal.

Measure ∠s DCO, BCO. They are equal.

Similarly ∠CDO = ∠ADO, ∠BAO = ∠OAD, ∠ABD = ∠CDB. In fact each of them is 45°.

Repeat this experiment with any other square. We conclude that

(*i*) all the sides of a square are equal,

(*ii*) all the angles are right angles,

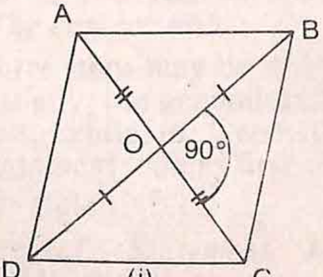
(*iii*) the diagonals are equal and they bisect each other at right angles,

(*iv*) the diagonals bisect the angles at the corners,

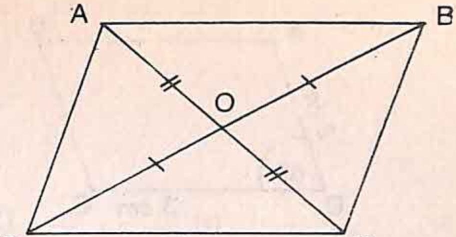
(*v*) each diagonal bisects the square.

Exercise 3

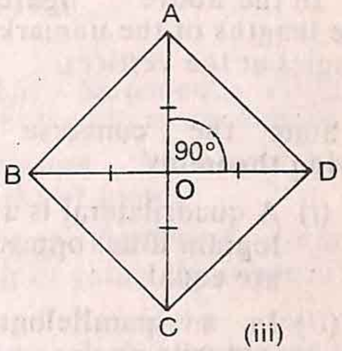
1. What type of figures are the following and why? (The lengths marked with similar signs are equal).



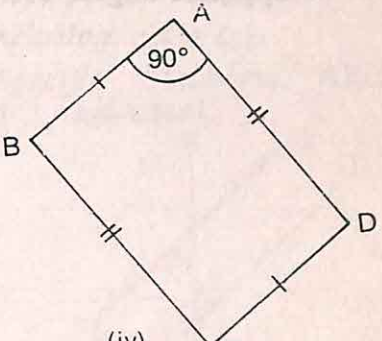
(i)



(ii)



(iii)



(iv)

2. Fill in the blanks:

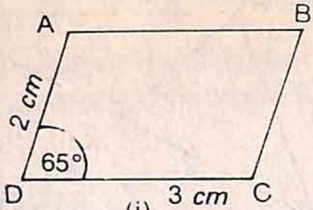
(*i*) The.........sides of a parallelogram are equal.

(*ii*) A quadrilateral is a.........if its opposite sides are equal.

(*iii*) The opposite sides of a parallelogram are......... and ....

(*iv*) The diagonals of a..... bisect each other at right angles but its angles are not right angles.

(*v*) Each......of a parallelogram bisects it.



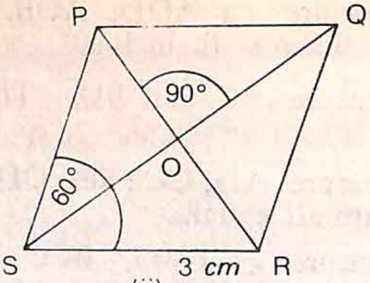
(i)

3. In the above figures, point out the lengths of the unmarked sides and angles at the vertices.

4. State the converse of the following theorems :

(*i*) A quadrilateral is a parallelogram if its opposite sides are equal.

(*ii*) In a parallelogram the opposite angles are equal.



(ii)

5. In a quadrilateral, the diagonals bisect each other and are both equal to 2·3 *cm* in length ; but they do not intersect at right angles. What kind of quadrilateral is this ?

6. In a quadrilateral the diagonals bisect each other at right angles but they are unequal in length ? What kind of quadrilateral is this ?

CHAPTER III

THEORETICAL GEOMETRY

1 So far all your conclusions were made by *inductive reasoning.* You observed many typical cases and noted the property which is common to all of them. You then *generalized* this common property which fitted all cases.

Now we are going to learn Geometry by *deductive reasoning.* This process is exactly opposite of inductive reasoning. It begins from the general and proceeds to the definite or specific. Here we start with the general statement and use this statement to a specific case.

For example, "*all dogs have curly tails*" is a general statement because it is about all dogs *in general.* And we know that "Tom is a dog having a curly tail" ; this is a specific statement, because it refers to a particular dog.

There are three steps to every deductive reasoning, namely : (1) *A general statement*, (2) *A specific statement*, satisfying all, the conditions of the general statement, and (3) *The conclusion.*

The three steps may be used in any order. Usually, the general statement comes first, while in Geometry the specific statement comes first.

For example :

1. *General Statement.* A boy must get 75% marks to get a merit scholarship in our school.

Specific Statement. Arun gets a merit scholarship in our school.

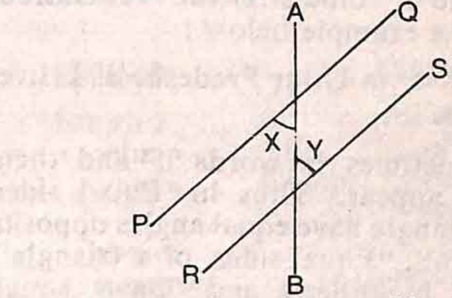
Conclusion. Arun must have got 75% marks or more.

2. *Specific Statement.* $\angle x$ and $\angle y$ are a pair of vertically opposite angles.

General Statement. Any pair of vertically opposite angles are equal.

Conclusion. $\angle x = \angle y$.

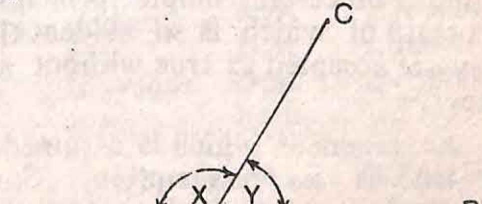
3. *Specific Statement.* Alternate $\angle x$ and $\angle y$ are equal.



General Statement. If alternate angles are equal, the straight lines are parallel.

Conclusion. PQ is parallel to RS.

4. *Specific Statement.* $\angle x$ and $\angle y$ are supplementary angles.



General Statement. If two adjacent angles are supplementary, then their exterior arms are in a straight line.

Conclusion. AO, OB are in a straight line.

II. Any general statement consists of two parts. The first part is the *supposition* and usually begins with 'if' This part is called the **hypothesis.** The second part which begins with 'then' is the **conclusion**. For example :

In the statement "if two straight lines intersect, then the vertically opposite angles are equal". "If two straight lines intersect" is the *hypothesis* and the *conclusion* is "then the vertically opposite angles are equal".

The hypothesis may come before the conclusion as in the above example or come after the conclusion as in the example below :

"I live in Uttar Pradesh, if I live in Agra."

Sometimes the words 'if' and 'then' do not appear. Thus in "Equal sides of a triangle have equal angles opposite to them", "Equal sides of a triangle" is the hypothesis and "have equal angles opposite to them" is the conclusion.

III. Assumptions, Axioms and Postulates

All mathematical reasoning is founded on certain simple principles, the truth of which is so evident that they are accepted as true without any proof.

A statement which is assumed to be true is an **assumption**. Some assumptions are called **axioms** and some are called **postulates**.

Axioms are self-evident truths. Thus an axiom is an obvious statement which cannot be proved by reference to any simpler statement.

A *postulate* is something that can be assumed *to be done* without any necessity of explanation of the manner in which it is done. In other words, a postulate is a problem so easy that it can be taken for granted.

We give below the most commonly used axioms in geometrical reasoning :

1. If equals are added to equals, the sums are equal.
2. If equals are taken from equals, the remainders are equal.
3. If equals are multiplied by equals, the products are equal.
4. If equals are divided by equals, the quotients are equal.
5. A quantity may be substituted for its equal.
6. Quantities which are equal to the same quantity are equal to each other.
7. If equals are added to unequals, the sums are unequal.
8. If equals are subtracted from unequals, the remainders are unequal.
9. The whole is equal to the sum of its parts and therefore, greater than any of its parts.
10. Two intersecting straight lines cannot both be parallel to the same straight line.

Examples of postulates are :

1. A straight line may be drawn from any point to any other point.
2. A finite straight line may be produced to any length in that line.
3. A circle may be drawn with any

point as centre and with any length as radius.

4. A geometrical figure may be moved from one position to another without change in form or size.

5. The study of Geometry consists of a series of **Propositions** – a number of *separate discussions*, each complete in itself, the latter ones depending on the earlier ones.

The propositions in geometry are of two kinds :

A **Theorem** is a proposition in which some geometrical property is to be *proved*. Thus the statement "*If two straight lines intersect, the vertically opposite angles are equal*" is a theorem.

A **Problem** is a proposition in which some geometrical construction is required to be *affected* ; for instance, "*Bisect a given angle*".

Every proposition consists of four parts :

1. The **general enunciation** is the statement in general terms what is to be proved or what is to be constructed.

2. The **particular enunciation** is the repetition of the general enunciation in special terms with reference to a particular diagram drawn for the purpose

3. The **construction** is the drawing of lines which may be necessary to prove the truth of a theorem or effect the object of a problem.

In theorems constructions *are not actually made*. They are taken for granted and are called **hypothetical constructions**, for example : draw AB perpendicular to BC at B.

4. The **proof** is that part of a proposition in which either the truth of a theorem is established or the object of a problem is shown to have been effected.

5. A corollary is a statement of truth which readily follows from a proposition already proved. It is, therefore, placed at the end of a proposition, and usually requires no further proof.

6. In **Theoretical Geometry** we prove the truth of some Geometrical statements by systematic reasoning.

In Practical Geometry we actually construct geometrical figures.

These things will be illustrated in the remaining part of the book.

7. The following symbols and abbreviations have been used for the purpose of economy of space and time :

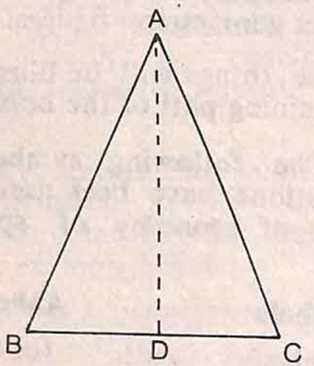
Symbols	
+	for *plus*
—	,, *minus*
∠	,, *angle*
∠s	,, *angles*
△s	,, *triangles*
⊥	,, *perpendicular to*
∥	,, *parallel to*
∥gm	,, *parallelogram*
○	,, *circle*
$\bigcirc^{ce}$	,, *circumference*
>	,, *is greater than*
<	,, *is less than*
∴	,, *therefore*

Abbreviations	
Adj.	for *adjacent*
Alt.	,, *alternate*
Compl.	,, *complement* or *Complementary*
Const.	,, *Construction*
Corresp.	,, *Corresponding*
Def.	,, *Definition*
Diag.	,, *Diagonal*
Diam.	,, *Diameter*
Equil.	,, *Equilateral*
Hyp.	,, *Hypothesis*
Isos.	,, *Isosceles*
Opp.	,, *Opposite*
Perp.	,, *Perpendicular*
Prob.	,, *Problem*
Prop.	,, *Proposition*

∵	*since*	Pt.	*Point*
BC^2	,, *square on BC*	Quad.	,, *Quadrilateral*
		Rect.	,, *Rectangle*
=	,, *equal to*	Sq.	,, *Square*
≡	,, *congruent*	St.	,, *Straight*

THEOREM 1

If two sides of a triangle are equal, the angles opposite to these sides are equal *i.e.*, the angles at the base of an isosceles triangle are equal.



Given :—In the $\triangle ABC$, $AB = AC$.

To prove :—$\angle B = \angle C$.

Const : - Draw AD, the bisector of $\angle A$, meeting BC in D.

Proof :—In the $\triangle$s ABD, ACD, $AB = AC$, $AD = AD$, and the contained $\angle BAD$ = contained $\angle CAD$

$\therefore \triangle ABD \equiv \triangle ACD$.

Hence $\angle B = \angle C$.

Cor. 1. If the equal sides AB, AC of an isosceles triangle are produced; the exterior angles so formed are equal, for they are the supplements of the equal angles at the base.

Cor. 2. An equilateral triangle is also equiangular.

Exercise 4

1. In the rhombus ABCD, prove that

(*i*) $\angle ABD = \angle ADB$,

(*ii*) $\angle DBC = \angle CDB$,

(*iii*) $\angle ABC = \angle ADC$.

2. The bisector of the vertical angle of an isosceles triangle bisects the base at right angles.

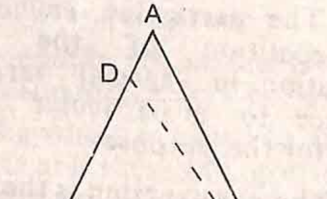
3. ABC, DBC are two isosceles triangles drawn on the same base BC; prove that $\angle ABD = \angle ACD$.

4. If the st. line joining A in the $\triangle ABC$ to the mid-point D of the base BC is equal to half BC, show that $\angle BAC = \angle B + \angle C$.

THEOREM 2

(Converse of Theorem 1)

If two angles of a triangle are equal, the sides opposite to these angles are equal.



Given :—In $\triangle ABC$, $\angle B = \angle C$.

To prove :—$AB = AC$.

Const. :—If AB is not equal to AC, cut off $BD = AC$. Join CD.

Proof. :—In the $\triangle$s DBC, ABC,

BD = AC (const.)
BC = BC.
$\angle$DBC = $\angle$ACB
(Hypothesis)

$\therefore$ $\triangle$DBC $\equiv$ $\triangle$ABC,
which is impossible.

[Part cannot be equal to the whole.]

$\therefore$ AB is not unequal to AC, *i.e.*,

AB = AC.

Cor. An equiangular triangle is also equilateral.

Exercise 5

1. The bisectors of the base angles B and C of a $\triangle$ABC meet in D so that DB = DC. Prove that the $\triangle$ABC is isosceles.

2. The exterior angles formed by producing two sides of a triangle are equal. Prove that the triangle is isosceles.

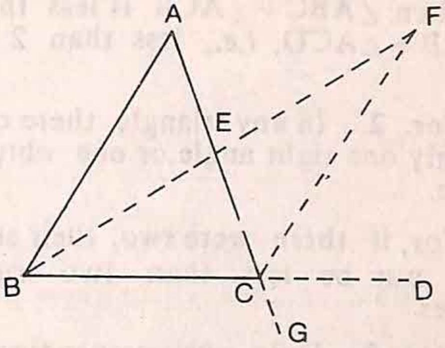
3. If the base of a triangle be produced both ways and the exterior angles thus formed are equal, prove that the triangle is isosceles.

4. A straight line perpendicular to the bisector of an angle forms an isosceles triangle with the arms of the angle.

THEOREM 3

If one side of a triangle is produced the exterior angle so formed is greater than either of the interior opposite angles.

Given :—The side BC of the $\triangle$ABC is produced to D.



To prove : —The ext. $\angle$ACD > either of the int. opposite $\angle$s BAC, ABC.

Const. —Join B to E, the middle point of AC.

Produce BE to F, so that EF = BE.
Join FC.

Proof :—In the $\triangle$s AEB, CEF,

CE = EF, AE = EC.
(Const.)
and $\angle$AEB = vert. opp. $\angle$CEF.

$\therefore$ $\triangle$AEB $\equiv$ $\triangle$CEF
(Two sides and included $\angle$)
$\therefore$ $\angle$ECF = $\angle$BAE.
i.e., $\angle$ACF = $\angle$BAC.
But $\angle$ACD > $\angle$ACF.
$\therefore$ $\angle$ACD > $\angle$BAC.

Similarly, by producing AC to G, it may be proved that the ext. $\angle$BCG > int. oppo. $\angle$ABC.

But $\angle$BCG = vert. oppo. $\angle$ACD

$\therefore$ $\angle$ACD > $\angle$ABC.

Cor. 1. Any two angles of a triangle are together less than two right angles.

Proof. ∠ABC is less than ∠ACD (proved)

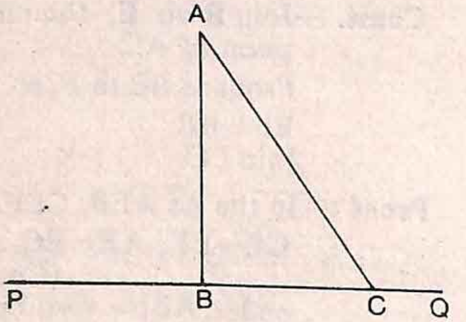
To each add the ∠ACB.

Then ∠ABC+∠ACB is less than ∠ACB+∠ACD, *i.e.*, less than 2 rt. ∠s.

Cor. 2. In any triangle there can be only one right angle or one obtuse angle.

For, if there were two, their sum will not be less than two right angles.

Cor. 3. Only one perpendicular can be drawn to a straight line from a given point outside it.



If two perps. AB, AC could be drawn to PQ from A, we would have a △ABC in which each of ∠s ABC, ACB would be a right angle which is impossible.

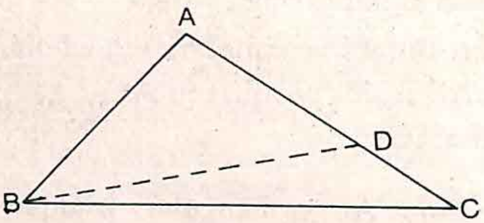
Exercise 6

1. If any side of a triangle is produced both ways, the exterior angles so formed are together greater than two right angles.

2. If the equal sides of an isosceles triangle are produced, the exterior angles so formed must be obtuse.

THEOREM 4

If one side of a triangle is greater than the other, then the angle opposite to the greater side is greater than the angle opposite to the lesser side.



Given :– In △ABC, AC>AB.

To prove :– ∠ABC>∠ACB.

Const. :—From AC cut off AD=AB.

Join BD.

Proof :—In △ABD, ∵ AB=AD,

∴ ∠ABD=∠ADB.

But the ext. ∠ADB of the △BDC is greater than the int. opp. ∠DCB, *i.e.*, greater than ∠ACB.

∴ ∠ABD>∠ACB

∴ ∠ABC is still more greater than ∠ACB.

Note. This theorem may also be stated thus :

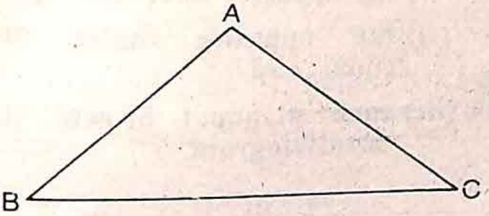
"If two sides of a triangle are unequal, the greater side has the greater angle opposite to it."

THEOREM 5

(Converse of Theorem 4)

If one angle of a triangle is greater than another, then the side opposite to the greater angle is

greater than the side opposite to the lesser angle.



Given :— In $\triangle ABC$, $\angle B > \angle C$.

To prove:— $AC > AB$.

Proof :—If AC is not $> AB$,
either (*i*) $AC = AB$, or
(*ii*) $AC < AB$.

But (*i*) if $AC = AB$ then
$\angle B = \angle C$,
which is not the case.
And (*ii*) if $AC < AB$ then
$\angle B < \angle C$,
which is also not the case.
Hence $AC > AB$.

Note. This theorem may also be enunciated thus :

"If two angles of a triangle are unequal the greater angle has the greater side opposite to it."

Exercise 7

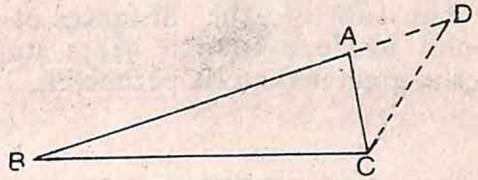
1. In a triangle the greatest angle has the greatest side opposite to it.

2. In a right-angled triangle the hypotenuse is the greatest side.

3. If D be a point on one of the equal sides AB of an isosceles $\triangle ABC$, show that $DC > DB$.

THEOREM 6

Any two sides of a triangle are together greater than the third.



Given :— A $\triangle$ ABC.

To prove :—(*i*) $AB + AC > CB$,
(*ii*) $AC + BC > AB$,
(*iii*) $AB + BC > AC$.

Const. :—Produce BA to D so that
$AD = AC$.
Join CD.

Proof :—In $\triangle ADC$, $\because AD = AC$,
$\therefore \angle ACD = \angle ADC$.
But $\angle BCD >$ its part $\angle ACD$.

$\therefore \angle BCD > \angle ADC$, *i e.*, $\angle BDC$.

$\therefore BD > BC$.
i.e., $AB + AC > BC$.

Similarly, $AC + BC > AB$ and $AB + BC > AC$.

Cor. The difference between any two sides of a triangle is less than the third side.

By subtracting AC from both sides, we get
$AB > BC - AC$.

Exercise 8

1. Each of the equal sides of an isosceles triangle is greater than half the base.

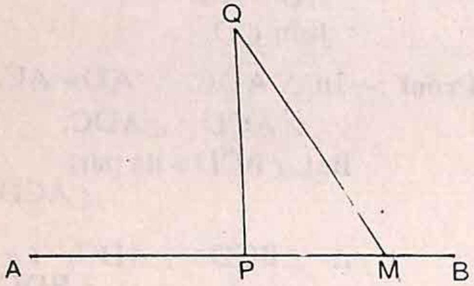
2. The semi-perimeter of a triangle is greater than any of its sides.

(Semi-perimeter means half the sum of its sides.)

3. The sum of the distances of any point inside a triangle from its vertices is greater than its perimeter.

THEOREM 7

Of all straight lines drawn to a given straight line from a given point outside it, the perpendicular is the least.



Given :—A straight line AB ; and Q a pt. outside it ; QP is drawn perp. to AB, and QM *any* other st. line from Q to the st. line AB.

To prove :—$QP < QM$.

Proof :—In $\triangle QPM$, the ext. $\angle QPA >$ the int. opp. $\angle QMP$.

But $\angle QPA = \angle QPM$, both being rt. $\angle$s

$\therefore \angle QPM > \angle QMP$.

$\therefore$ in $\triangle QPM$, $QP < QM$.

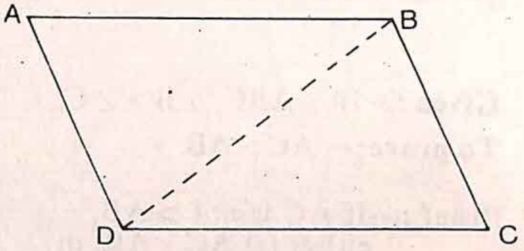
THEOREM 8

In a parallelogram

(*i*) the opposite sides are equal,

(*ii*) the opposite angles are equal, and

(*iii*) each diagonal bisects the parallelogram.



Given :—A parallelogram ABCD.

To prove :—(*i*) $AB = DC$, $AD = BC$.

(*ii*) $\angle A = \angle C$, $\angle B = \angle D$

(*iii*) BD and AC each bisects the parallelogram ABCD.

Const. :—Join BD.

Proof :—$\because AB \parallel DC$ and BD cuts them,

$\therefore \angle ABD =$ alt. $\angle BDC$.

Again $\because AD \parallel BC$ and BD cuts them.

$\therefore \angle ADB =$ alt. $\angle DBC$.

Hence in the $\triangle$s ABD and BCD,

$\because$ $\angle ABD = \angle BDC$, $\angle ADB = \angle DBC$ (*proved*) and $BD = BD$.

$\therefore \triangle ABD \equiv \triangle BDC$ (two angles and one side of one $\triangle =$ two angles and one side of the other $\triangle$)

$\therefore AB = DC$, and $AD = BC$. ...(*i*)

Also $\angle A = \angle C$.

Similarly, by joining AC, we can prove

$\angle B = \angle D.$ ...(*ii*)

And since $\triangle ABD \cong \triangle BDC$,

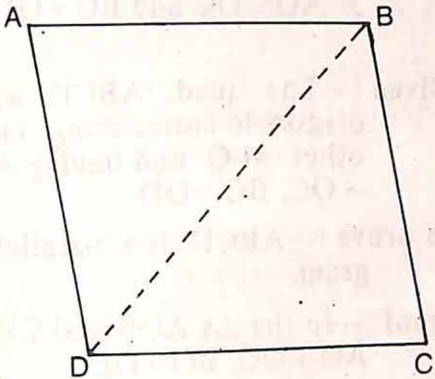
$\therefore$ BD bisects the ∥ gm. ABCD.

Similarly, AC bisects the ∥ gm. ABCD. ....(*iii*)

THEOREM 9

A quadrilateral is a parallolgram if

- **(*i*) the opposite sides are equal, or**
- **(*ii*) the opposite angles are equal, or**
- **(*iii*) one pair of opposite sides are equal and parallel.**



(*i*) **Given** :—A quad. ABCD, in which $AB = CD$. $AD = BC$.

To prove :—ABCD is a ∥ gm.

Const. :—Join BD.

Proof :–In $\triangle$s ABD, BCD,

$$\because \begin{cases} AB = CD, \\ AD = BC, \\ BD = BD, \end{cases}$$

$\therefore \triangle ABD \cong \triangle BCD$. (three sides of one $\triangle$ = three sides of another $\triangle$)

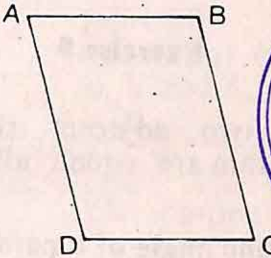
$\therefore \angle ABD = \angle BDC$, and $\angle ADB = \angle DBC$.

But these are pairs of alternate angles,

$\therefore$ AB ∥ DC, and AD ∥ BC.

Hence ABCD is a ∥ gm.

(*ii*) **Given** :–A quad. ABCD in which $\angle A \;\; \angle C$, $\angle B = \angle D$.



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To prove :–ABCD is a ∥ gm.

Proof :–$\angle A + \angle B + \angle C + \angle D = 4$ rt. $\angle$s.

or $2\angle A + 2\angle B = 4$ rt. $\angle$s

$\therefore \angle A + \angle B = 2$ rt. $\angle$s.

But these are interior $\angle$s on the same side of the cutting line AB.

$\therefore$ AD ∥ BC.

Similarly AB ∥ DC.

Hence ABCD is a ∥ gm.

Note. These two parts (*i*) and (*ii*) are the converse of parts (*i*) and (*ii*) of the earlier theorem.

(*iii*) **Given** :–A quad. ABCD, in which AB and CD are equal and parallel.
(See Fig. of theorem 9)

To prove :–ABCD is a ∥ gm.

Const :–Join BD.

Proof :–$\because$ AB and DC are parallel and BD cuts them,

$\therefore \angle ABD =$ alt. $\angle BDC$.

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Now in $\triangle$s ABD, BCD

$\because AB=DC, BD=BD,$

and contained $\angle ABD$=contained $\angle BDC$,

$\therefore \triangle ABD \equiv \triangle BCD.$

$\therefore \angle ADB = \angle CBD.$

But these are alternate $\angle$s.

$\therefore AD \parallel BC.$

Hence ABCD is a $\parallel$ gm.

Exercise 9

1. If two adjacent sides of a parallelogram are equal, all its sides are equal.

2. If one angle of a parallelogram is a right angle, all its angles are right angles

3. Parallel straight lines are everywhere equidistant.

4. The bisectors of two adjacent angles of a parallelogram are at right angles, and the bisectors of two opposite angles are parallel.

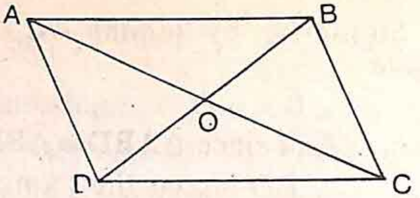
5. ABCD and ABEF are two parallelograms on the same base AB and on the same side of it. Show that CDEF is a parallelogram.

[**Hint.** Lines AB, FE and CD are equal and parallel.]

THEOREM 10

(*i*) The diagonals of a parallelogram bisect each other.

(*ii*) *Conversely*. If the diagonals of a quadrilateral bisect each other, it is a parallelogram.



(*i*) **Given** :—The $\parallel$ gm. ABCD, with diagonals intersecting each other at O.

To prove :—$AO=OC$ and $BO=OD$.

Proof :—In $\triangle$s AOB and COD,

$AB=CD$, (opp. sides of a $\parallel$ gm.)

$\angle AOB$ vert. opp. $\angle COD$,

and $\angle ABO$=alt. $\angle ODC$.

$\therefore \triangle AOB \equiv \triangle COD$. (Two angles and one side of one $\triangle$=two $\angle$s and one side of the other $\triangle$)

$\therefore AO=OC$ and $BO=OD$.

(*ii*) **Given** : –The quad. ABCD with diagonals intersecting each other at O and having $AO=OC$, $BO=OD$.

To prove :—ABCD is a parallelogram.

Proof :—In the $\triangle$s AOB and COD.

$AO=OC$, $BO=OD$.

$\angle AOB$=vert. opp. $\angle COD$.

$\therefore \triangle AOB \equiv \triangle COD$. (Two sides and the included $\angle$ of one $\triangle$=two sides and the included $\angle$ of the other $\triangle$.)

$\therefore \angle ABO = \angle ODC$.

But these are alternate angles.

$\therefore AB \parallel DC$.

Similarly, from △s AOD, BOC, it may be shown that AD ∥ BC.

∴ ABCD is a parallelogram.

Exercise 10

1. ABCD is a parallelogram ; prove that the perpendiculars AE and CF on the diagonal BD are equal.

2. If the diagonals of a quadrilateral bisect one another at right angles, it is either a square or a rhombus.

3. If the diagonals of a quadrilateral bisect one another at right angles and are equal, the figure is a square.

4. Prove that a parallelogram is :

(*i*) a rectangle, if its diagonals are equal ;

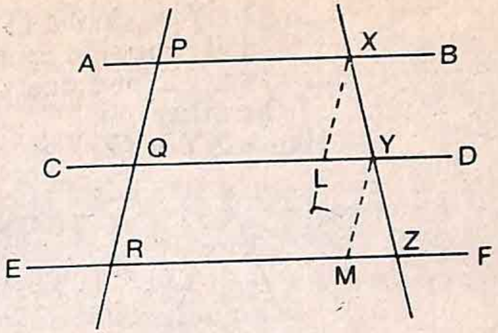
(*ii*) a square, if its diagonals are equal and perpendicular.

5. Prove that the bisectors of the angles of a parallelogram enclose a rectangle.

THEOREM 11

If three or more parallel straight lines make equal intercepts on a transversal, they make equal intercepts on any other transversal.

Given :—Three st. lines AB, CD, EF are cut by a transversal at P, Q and R so that PQ=QR ; another transversal cuts them at X, Y and Z.



To prove :—XY = YZ.

Const :—Draw XL, YM each ∥ PR meeting CD, EF in L and M, respectively.

Proof :—∵ XL ∥ PR and YM ∥ PR,
∴ XL ∥ YM (st. lines which are to the same st. line are ∥ to one another).

Now, since XL ∥ YM, and YZ cuts them,

∴ ∠LXY corresp. ∠MYZ.

Again, ∵ CD ∥ EF, and XZ cuts them,

∴ ∠XYL = corresp. ∠YZM.

Now PX ∥ QL and PQ ∥ XL,

∴ PQLX is a ∥ gm.

∴ XL = PQ.

Also QRMY is a ∥ gm.

∴ YM ∥ QR

But PQ=QR.

∴ XL = YM.

In the △s XLY, YMZ, ∠LXY = ∠MYZ, ∠XYL = ∠YZM (*proved*) and XL = YM (*proved*).

Const. : Produce ED to F so that $DF = DE$. Join FB.

Proof :—$DE = DF$, $AD = DB$, $\angle ADE = \text{vert. opp. } \angle BDF$.

$\therefore \triangle ADE \equiv \triangle FDB$. (Two sides and included $\angle$).

$\therefore BF = AE = EC$, and $\angle EAD = \angle DBF$

But these are alternate $\angle$s.

$\therefore AE$, *i.e.*, $CE \parallel BF$

$\therefore BCEF$ is a $\parallel$ *gm.*

$\therefore DE \parallel BC$.

Also $DE = \frac{1}{2}FE = \frac{1}{2}BC$.

3. The st. lines joining the mid-points of the sides of a triangle divide it into four congruent triangles.

4. The st. line which joins the mid-points of two adjacent sides of a quadrilateral is parallel to a diagonal, and equal to half the diagonal.

5. The st. lines joining the middle points of the adjacent sides of a quadrilateral form a parallelogram.

[**Hint.** Apply Ex. 5.]

6. Prove that the figure formed by joining the middle points of the adjacent sides of a rectangle is a rhombus.

7. The st. lines joining the middle points of the opposite sides of a quadrilateral bisect one another.

[Hint. They are the diagonals of the parallelogram formed in Ex. 6.]

AREAS

8. You know what a rectangle is. The page of this book, the faces of a brick, the floor of a room are all rectangles, for their opposite sides are equal and their angles are right angles.

The sides of a rectangle are usually called its **length** and **breadth.** In the *rectangle* the length and breadth are unequal. In the *square* the length is equal to the breadth.

The **perimeter** of a rectangle is the sum of all its sides.

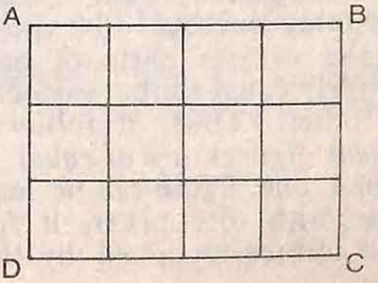
9. The **unit of area** is the area of a square whose side is a unit of length.

If the unit of length is a *metre*, the unit of area is a *square metre* ; if the unit of length is a *centimetre*, the unit of area is a *square centimetre*.

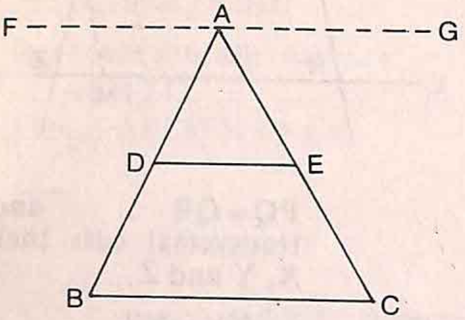
The terms *square decimetre square dekametre*, and *square kilometre*, are used in the same sense.

10. **Measurement of area.** To *measure* an area is to find the *number of times* it contains the unit area or *what part* it is of the unit area.

11. **Area of a rectangle.** Take a rectangle 4 *cm.* long and 3 *cm.* broad. Draw parallels as in the figure. The



$\therefore$ $\triangle XLY \equiv \triangle YMZ$ (Two $\angle$s and one side of one $\triangle$ = two $\angle$s and one side of the other $\triangle$).
Hence $XY = YZ$.



Cor. **The straight line drawn through the middle point of one side of a triangle parallel to the second bisects the third.**

Given :—A $\triangle ABC$, and DE drawn parallel to BC from D, the middle point of AB.

To prove :— $AE = EC$.

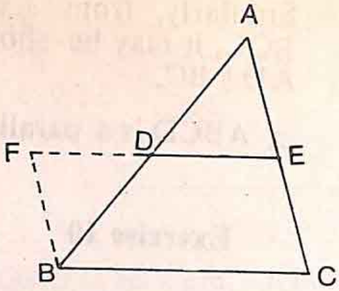
Const. :—Through A draw FAG $\parallel$ BC.

Proof :—$\because$ parallel st. lines BC, DE, FG make equal intercepts AD and DB on the transversal AB,
$\therefore$ they must make equal intercepts on the transversal AC also.
$\therefore$ $AE = EC$.

Exercise 11

1. The straight line drawn through the middle point of one side of a triangle parallel to the base, bisects the other side and is equal to half the base.

[The first part has already been proved above ; but we give here a different proof.]



Given :—A $\triangle ABC$ and DE drawn parallel to BC from D, the mid-point of AB.

To prove :—$AE = EC$, and $DE = \frac{1}{2}BC$.

Const. :—Produce ED to F so that $DF = ED$.
Join FB.

Proof :—In the $\triangle$s ADE, FDB,
$AD = BD$, $DE = FD$, $\angle ADF$ vert. opp. $\angle BDF$.
$\therefore$ $\triangle ADE \equiv \triangle FDB$. (Two sides and the included $\angle$ of one $\triangle$ = two sides and included $\angle$ of another).
$\therefore$ $\triangle ADE \equiv \triangle FDB$.
$\therefore$ $AF = BF$, and $\angle DAE = \angle DBF$ which are alternate $\angle$s.
$\therefore$ $BF \parallel CE$. But $EF \parallel CB$.
$\therefore$ BCEF is a $\parallel$ gm.
Hence $BF = CE$; $EF = BC$.
But $BF = AE$; $\therefore AE \doteq CE$.
Also $DE = \frac{1}{2}EF \doteq \frac{1}{2}BC$.

2. The straight line joining the mid-points of any two sides of a triangle is parallel to the third and is half of it.

[For figure see Ex. 1]

Given :—$\triangle ABC$ in which D and E are mid-points of AB and AC.

To prove :—$DE \parallel BC$ and $DE = \frac{1}{2}BC$.

student will have no difficulty in seeing that the small quadrilaterals are squares with each side equal to 1 *cm*.

In each row there are 4 small squares and there are 3 such rows.

∴ The area of the rectangle is 4×3 or 12 sq. *cm*. A similar result holds for any numbers, which may be chosen.

Thus, **area of a rectangle = Length × Breadth.** Area of a rectangle of sides a and b units $= ab$ sq. units. Area of a square of side a units $= a^2$ sq. units.

Note. It follows from the above result that :

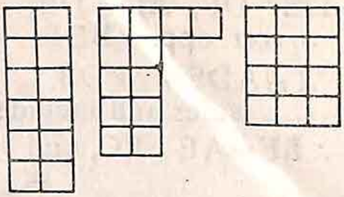
(*i*) Rectangles which have equal lengths and equal breadths have equal areas, and

(*ii*) Rectangles which have equal areas and equal lengths or breadths have equal breadths or lenghts.

12. Notation. In the figure of Art. 10 the rectangle is said to be *contained* by two adjacent sides such as AB and AD, and the *area* may be written AB.AD.

A rectangle ABCD whose adjacent sides are AB, AD, is denoted by rect. AB.AD or rect. AC (by taking two letters at the diagonally opposite vertices).

13. Congruency and Equality Two figures are said to be *congruent* when the various parts of one are respectively equal to the various parts of the other. Thus, it follows that *congruent figures are of equal area*; for, when one figure can be made to coincide with the other, it follows that the surface contained by the one will coincide with the surface contained by the other.



It should be noted, however, that figures which are not congruent may be equal in area, as the diagrams here show.

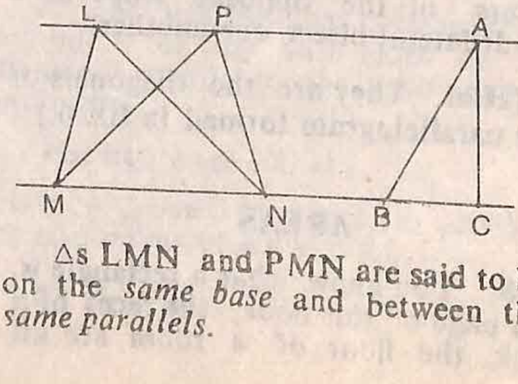
In all the three figures, the area = 12 small equal squares, but they are not congruent.

Figures which are equal in area only are said to be **equivalent.**

When two triangles are not congruent but *equivalent*, we use the notation $\triangle ABC = \triangle DEF$, and not $\triangle ABC \equiv \triangle DEF$.

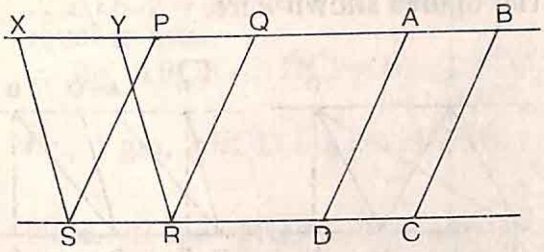
All congruent figures are equivalent in area, but equivalent figures are not necessarily congruent.

14. Between the same parallels. Triangles are said to be **between the same parallels** when their bases lie on one of the two parallel straight lines and their vertices on the other, as in the case of ∆s MPN and ABC.



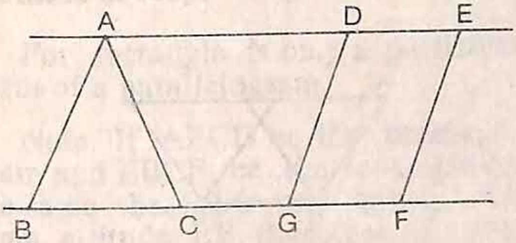
∆s LMN and PMN are said to be on the *same base* and between the *same parallels*.

Parallelograms are said to be **between the same parallels** when a pair of opposite sides of each lie on two parallel straight lines, as the || gms. PQRS and ABCD.



Parallelograms XYRS and PQRS are said on the *same base and between the same parallels.*

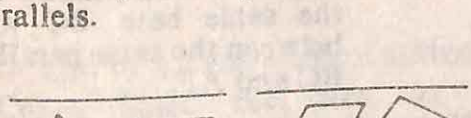
A triangle and a parallelogram are said to be between the same parallels when their bases are in the



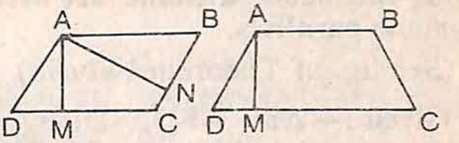
same straight line, and the side of the parallelogram opposite the base, produced, if necessary, passes through the vertex of the triangle, as

△ABC and the || gm. DEFG.

Caution. We do not speak of the two △s or of the two || gms. in the figure here, as being between the same parallels.



15. The altitude (or height) of a *parallelogram* or a *trapezium* is the perpendicular distance between a pair of parallel sides.



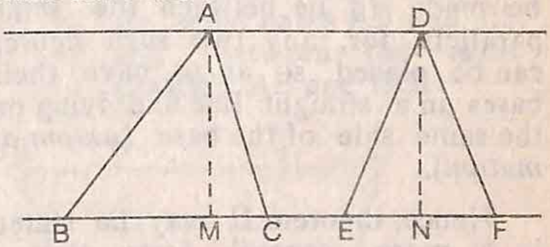
Thus, a parallelogram will have two altitudes, and a trapezium only one.

AM is the altitude when DC is the base, AN is the other altitude of the || gm., when BC is taken as the base.

How many altitudes are there in a triangle ? Three.

16. Two Theorems.

I. Triangles (or parallelograms) between the same parallels are of the same altitude.



Given :—Two △s ABC, DEF between the same parallels AD and BF.

To prove :—AM=DN.

Const. Draw AM, DN ⊥s BF, produced, if necessary.

Proof : ∵ AM, DN are both ⊥ BF,

∴ ∠AMN + ∠DMN = 2 rt. ∠s

∴ AM || DN.

∴ AMND is a || gm.

Hence AM=DN.

11. Triangles (or parallelograms) having their bases in a st. line and lying on the same side of the base and having the same altitude are between the same parallels.

(See fig. of Theorem I above)

Given :— $\triangle$s ABD, DEF with equal altitudes AM, DN and bases BC, EF in the same st. line BF, and their vertices A, D on the same side of BF

To prove : —AD ∥ BF.

Proof : —∵ $\angle$AMN+DNM=2 rt. $\angle$s.

∴ AM ∥ DN, and also AM=DN.

∴ AMND is a ∥ gm.

∴ AD ∥ MN or BF.

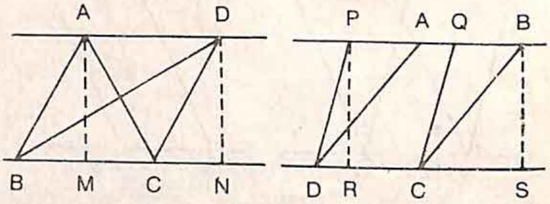
Note 1. Triangles (or parallelograms) having the same altitude can be made to lie between the same parallels, for, any two such figures can be placed so as to have their bases in a straight line and lying on the same side of the base (*axiom of motion*).

Hence, theorem II may be stated in a more general form thus : "*Triangles (or parallelograms) having the same altitude are between the same parallels.*"

This is converse of theorem I above. It follows, therefore, that the phrase "lying between the same parallels", is equivalent to the phrase "*having the same altitude*".

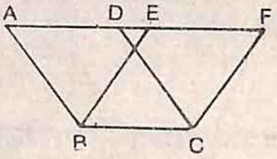
Note 2. The student should, as an exercise, make the corresponding figures for the case of *two parallelograms* instead of two triangles in the above two theorems.

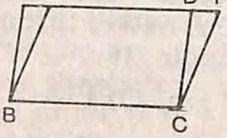
Note 3. Again if two $\triangle$s or two parallelograms have equal bases and the same altitude, they can always be placed where they stand on the same base and have the same altitude as the figures shown here.

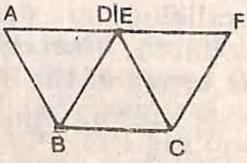


THEOREM 12

Parallelograms on the same base and between the same parallels (or of the same altitude) are equal in area.







Given :— ∥ gms. ABCD, EBCF on the same base BC and between the same parallels BC and AF.

To prove :— ∥ gm. ABCD = ∥ gm. EBCF.

Proof :—In all the three figures,

∵ BA, BE are resp. ∥ to CD, CF

∴ $\angle$ABE = $\angle$DCF.

Now, in the △s DCF, ABE,

$$\because \begin{cases} DC=AB \text{ (Opp. sides of } \| \text{ gms.)} \\ FC=EB \\ \text{Contained } \angle DCF = \text{contained } \angle ABE \text{ (proved).} \end{cases}$$

∴ △DCF ≡ △ABE and therefore. equal in area.

∴ fig. ABCF − △DCF = fig. ABCE − △ABE,

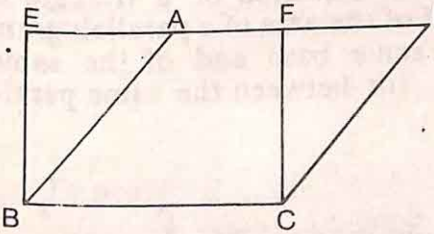
i.e., || gm. ABCD = || gm. EBCF.

Note. If the given || gms. are of the *same altitude*, they are between the same parallels (Art. 16), and the same proof will apply.

Cor 1. The area of a parallelogram is equal to that of a rectangle on the same base and having the same altitude.

For rectangle is only a particular form of a parallelogram.

Note. If ABCD be the parallelogram and EBCF be the rectangle on the same base BC and having the same altitude BE, then area of || gm. ABCD = area or rect. EBCF = BC.CF.



∴ area of a || gm. = base × altitude.

Cor. 1. Since the area of a parallelogram depends only on the base and altitude, it follows that—

Parallelograms on equal bases and of equal altitudes (or between the same parallels) are equal in area.

For a different proof, see Ex. 4 below.

Exercise 12

1. (*a*) Find the area of a parallelogram whose base is 6·5 *cm* and altitude 8 *cm*.

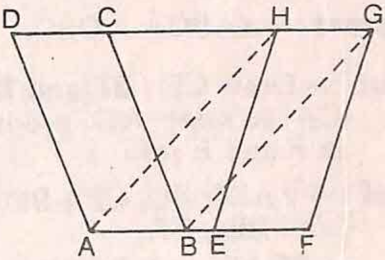
(*b*) Find the altitude of a parallelogram whose area is 8·4 sq. *cm* and base is 1·4 *cm*.

2. Prove that equal parallelograms on equal bases have the same altitude.

3. Prove that equal parallelograms having the same altitude stand on equal bases.

4. Prove that parallelograms on equal bases and between the same parallels (or of equal altitude) are equal in area.

Given :—Two || gms. ABCD, EFGH on equal bases AB and EF and between the same parallels AF and DG.



To prove :— || gm. ABCD = || gm. EFGH.

Const. :—Join AH and BG.

Proof :—AB = EF = HG.
Also AB || HG.

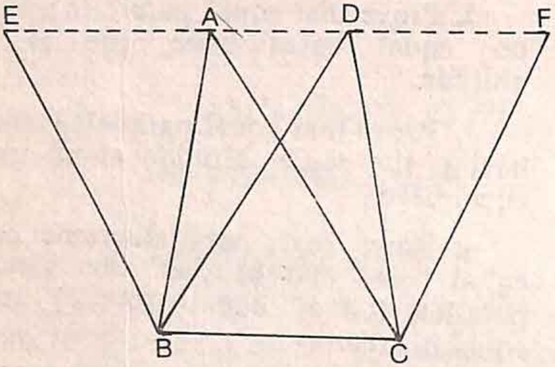
∴ ABGH is a || gm.

∴ || gm. ABCD = || gm. ABGH
= || gm. EFGH.

5. Prove that the st. lines joining the middle points of the sides of a triangle form with those sides three equal parallelograms.

THEOREM 13

Triangles on the same base and between the same parallels are equal in area.



Given :—Two △s ABC, DBC on the same base BC and between the same ∥ s AD and BC.

To prove :—△ABC= △DBC.

Const. :—Draw CF ∥ BD and BE ∥ CA, to meet AD produced in F and E resp.

Proof :—∵ AD ∥ BC, CF ∥ BD and BE ∥ CA.

∴ BCAE and BCFD are ∥ gms.

∵ AB is the diag. of ∥ gm. BCAE.

∴ △ABC=$\frac{1}{2}$ ∥ gm. BCAF.

∵ CD is a diag. of ∥ gm. BCFD.

∴ △DBC=$\frac{1}{2}$ ∥ gm. BCFD.

But ∥ gms. BCAE and BCFD stand on the same base BC and are between the same ∥ s BC and EF.

∴ ∥ gm. BCAE= ∥ gm. BCFD.

∴ △ABC= △DBC.

Cor. 1. Triangles on the same base and of the same altitude are equal in area.

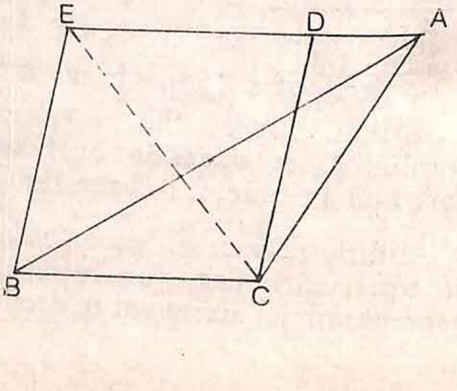
Since they are of the same altitude, they are between the same parallels.

Cor. 2. Triangles on equal bases and of the same altitude (or between the same parallels) are equivalent.

For they can be so placed as to be on the *same base and between the same parallels*.

[Also see note to Cor. 5 further on].

Cor. 3. The area of a triangle is one half of the area of a parallelogram on the same base and of the same altitude (or between the same parallels).



Given :— $\triangle ABC$ and the ∥ gm. BCDE on the same base BC and between the same parallels.

To prove :—$\triangle ABC = \frac{1}{2}$ ∥ gm. BCDE.

Const :—Join the diag. BC.

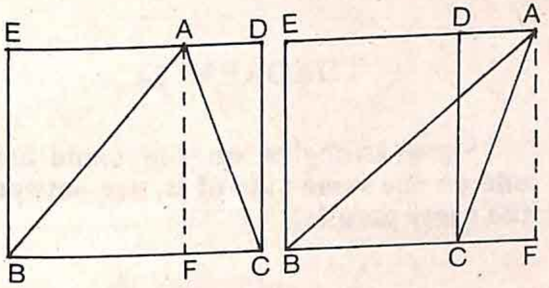
Proof :— $\triangle ABC = \triangle EBC$.
($\triangle$s on the same base and between the same parallels)
$= \frac{1}{2}$ ∥ gm. BCDE.

Cor. 4. The area of a triangle is half the area of the rectangle on the same base and of the same altitude.

The result follows from Cor. 3.

Alternative Proof :

Given :—$\triangle ABC$ and rect. BCDE on the same base BC and of the same altitude.



To prove :—$\triangle ABC = \frac{1}{2}$ rect. BCDE.

Const. :—Draw $AF \perp BC$ or BC produced.

Proof : – $\triangle AFB = \frac{1}{2}$ rect. EF, $\triangle AFC = \frac{1}{2}$ rect. FD.

Adding in fig. 1 and taking the difference in fig. 2, we get $\triangle ABC = \frac{1}{2}$ rect. BCDE.

Cor. 5. **Area of a $\triangle = \frac{1}{2}$ base × altitude.**

$$\therefore \text{ altitude} = \frac{\text{twice area}}{\text{base}};$$

$$\text{and, base} = \frac{\text{twice area}}{\text{altitude}}.$$

Note. Since the area of a $\triangle$ is dependent on its base and altitude, it follows that :

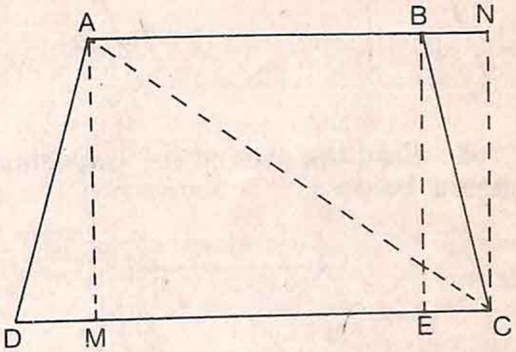
Triangles on equal bases and of equal altitudes are equal in area. (Cor. 2).

Area of a Trapezium

The area of a trapezium is equal to half the sum of the parallel sides multiplied by the altitude.

Given :—Trap. ABCD, where AB and DC are parallel and $BE \perp DC$.

To prove : - Area of trap. ABCD $= \frac{1}{2}(CD + AB).BE$.



Const. :—Join AC. Draw $AM \perp DC$, $CN \perp AB$, produced.

Proof :—Trap. ABCD
$= \triangle DAC + \triangle ABC$
$= \frac{1}{2}AM.CD + \frac{1}{2}AB.CN.$
$= \frac{1}{2}BE.CD + \frac{1}{2}BE.AB$
$(\because AM = BE = CN)$
$= (CD + AB).BE,$

i.e., area of the trap. = half the sum of the parallel sides × the altitude.

Exercise 13

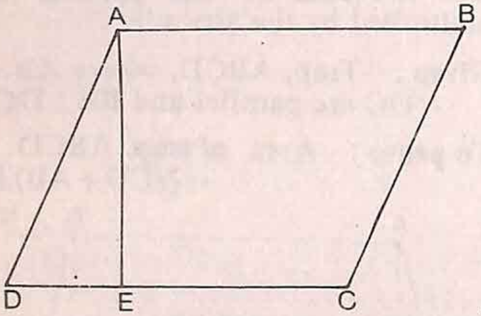
1. Find the areas of the following triangles :

(*i*) Base=2·5 *cm*, altitude=1·6 *cm*.

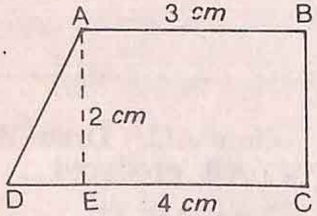
(*ii*) Base= 3 *cm*, altitude=1·8 *cm*.

2. Find the altitude of a triangle whose area is 4·8 sq. *cm* and base 1·2 *cm*.

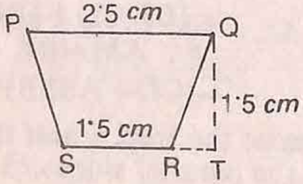
3. Find the area of the triangle DBC in the figure shown below if AB=3 *cm* and AE=2 cm.



4. Find the area of the trapeziums shown below :



(i)



(ii)

5. Find the area of the trapezium ABCD shown above, if AB=5 *cm*, AE =2 *cm*, area of △ADE=2 sq. *cm*, and EC=6 *cm*.

6. Prove that the straight line joining the vertex of a triangle to the middle point of the base (*i.e.*, the median) divides the triangle into two equal parts.

7. Prove that the area of a rhombus is half the product of its diagonals.

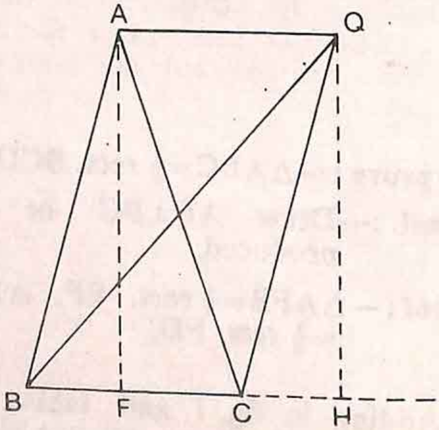
8. Prove that a parallelogram is divided into four equal parts by its diagonals.

9. How would you divide a triangle into three equal parts ?

10. P is any point on the median AD of the △ ABC ; show that △ ABP = △ ACP.

THEOREM 14

Equal triangles on the same base and on the same side of it, are between the same parallels.



Given : Equal △s ABC, QBC on the same base BC.

To prove :— $AQ \parallel BC$.

Const. : - Draw AF. QH perps. to BC, produced, if necessary.

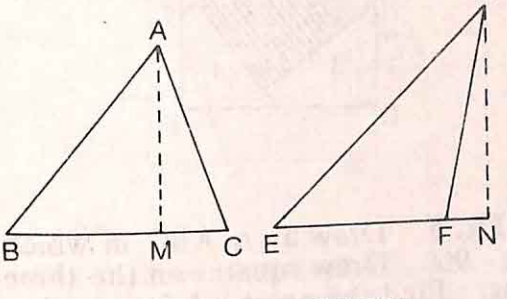
Proof :— $\triangle ABC = \frac{1}{2} AF.BC$, and
$\triangle QBC = \frac{1}{2} QH.BC$.
$\therefore \frac{1}{2} AF.BC = \frac{1}{2} QH.BC$.
$\therefore AF = QH$.
Also AF and QH are parallel.
Hence AQ and FH are parallel.

Cor 1. Equal triangles on the same base and on the same sides of it are of the same altitude.

For $AF = QH$, as proved above.

Cor. 2. Equal triangles on equal bases have equal altitudes.

[Hint. $\triangle ABC = \frac{1}{2} BC.AM$
$\triangle DEF = \frac{1}{2} FE.DN$

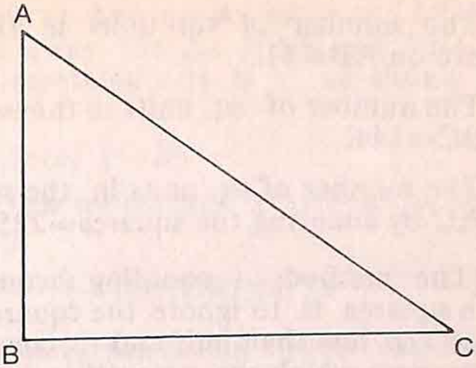


$\therefore \quad \frac{1}{2} BC.AM = \frac{1}{2} EF.DN$
But $\quad BC = EF$.
$\therefore \quad AM = DN$.]

Cor. 3. Equal triangles with equal altitudes are on equal bases.

Pythagoras Theorem

Ex. 1. Draw a $\triangle ABC$ in which $\angle B = 90°$, $BC = 4$ *cm* and $AB = 3$ *cm*. Measure AC. If the $\triangle$ has been drawn carefully, AC will be 5 *cm*. The area of the square of $AB = 3^2$ sq. *cm*. The area of the square on $BC = 4^2$ sq. *cm*.



$\therefore$ the sum of the squares on AB and $BC = 3^2 + 4^2 = 25$ sq. *cm*. The area of the square on the hypotenuse $AC = 5^2$ sq. *cm*.

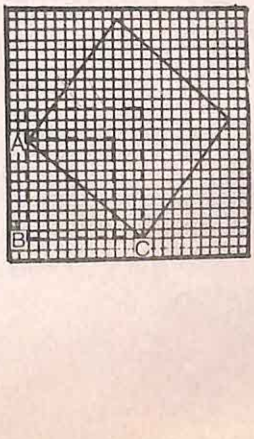
Hence, we see that $AB^2 + BC^2 = AC^2$.

Ex. 2. Repeat the experiment with $\triangle$s whose sides are (*i*) 6 *cm* and 8 *cm*, (*ii*) 5 *cm* and 12 *cm*, (*iii*) 8 *cm* and 15 *cm*.

In (*i*) AC will be 10 cm, in (*ii*) AC will be 13 *cm* and in (*iii*) AC will be 17 *cm*.

Thus, the relation $AB^2 + BC^2 = AC^2$ will be confirmed.

Ex. 3. On a squared paper draw a right-angled triangle ABC in which $AB = 9$, $BC = 12$. Draw squares on AB, BC, CA as shown. Count the number of square units in the squares on AB, BC and CA.



The number of sq. units in the square on AB=81.

The number of sq. units in the sq. on BC=144.

The number of sq. units in the sq. on AC by counting the squares=225.

[The method of counting incomplete squares is to ignore the squares which are less than half and to count the squares which are more than half as complete squares.]

$\therefore\ AB^2+BC^2=AC^2$.

This is known as the Theorem of Pythagoras.

In words we say :

In any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Ex. 4.

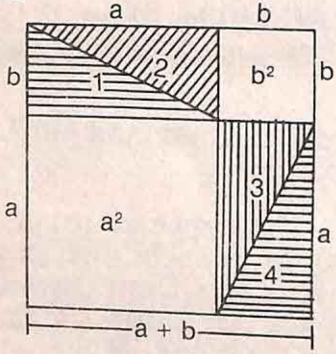


Fig. 1.

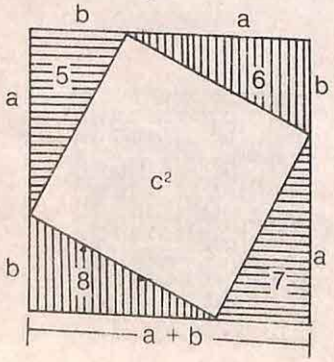
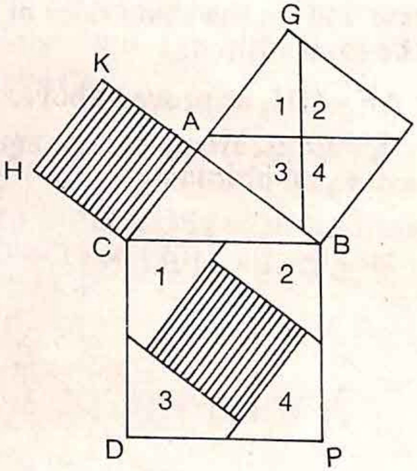


Fig. 2.

In the above figures, two squares have been shown. They are constructed on a length $=a+b$ units. Four congruent triangles have been cut off from these squares in two different ways. The *remaining* area in the Fig. $1=(a^2+b^2)$ square units ; and the remaining area in Fig. $2=c^2$. Hence, we have $a^2+b^2=c^2$.



Ex. 5. Draw a △ ABC in which ∠A = 90°. Draw squares on the three sides. Find the point of intersection of the diagonals of the square on the bigger side AB. Through this point draw lines parallel and perpendicular to the hypotenuse BC. Now the square on AB has been divided into four *congruent* quadrilaterals.

These, together with the square on AC, will be found *exactly* to fit into the square on BC, in the way indicated by corresponding numbers.

Thus we conclude that $AB^2+AC^2=BC^2$.

Ex. 6. Suppose we have a rectangle whose sides are a and b, and its diagonal is c. Draw a straight line $ABC=(a+b)$ long. Draw a square ABFH on AB and the square BCDE on BC. Cut off AK = BC and produce BF to G so that FG = BC. Join HK, KD, DG and GH.

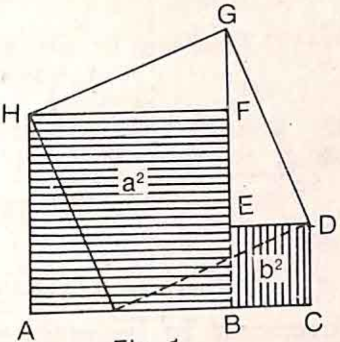


Fig. 1.

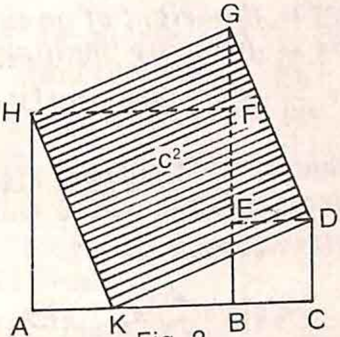


Fig. 2.

Now the △s HGF, HAK, DKC and GED thus formed are congruent, as can be easily seen ; and the figure HKDG is a square $=c^2$.

From the figure ACDGH, first take away the △s HGF and GED, the remaining part is a^2+b^2, as shown in Fig. 1.

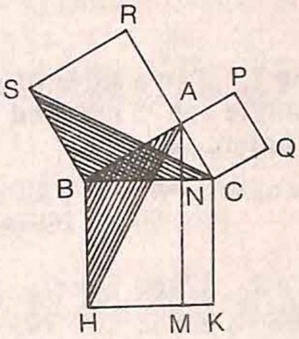
From the same figure ACDGH, take away the △s HAK and DKC ; the remaining part is c^2 as shown in Fig. 2.

Hence $a^2+b^2=c^2$.

Now we proceed to give a *formal proof*.

THEOREM 15

The square on the hypotenuse of a right-angled triangle is equal to the sum of the squares on the other two sides.



Given :—A △ABC, right-angled at A.

To prove :— $BC^2=AB^2+AC^2$.

Const. :—Describe sqs. BK, AQ, AS on BC, CA, AB resp. From A draw AM ∥ BH meeting HK in M. Join AH and CS.

Proof :—∠RAB, ∠BAC and ∠CAP are rt. ∠s.

∴ BAP and CAR are st. lines.

In △s ABH and SBC, AB = SB, BH = BC, and the contained ∠ABH = 90° + ∠B = contained ∠SBC.

∴ △ABH ≡ △SBC.

But △ABH = ½ rect. BM ; (on the same base and between the same ∥ s), and △SBC = ½ sq. AS ; (on the same base and between the same ∥ s),

∴ sq. AS = rect. BM.

Similarly, by joining AK and BQ, we can prove that sq. AQ = rect. CM.

∴ rect. BM + rect. CM = sq. AS + sq. AQ.

or sq. BK = sq. AS + sq. AQ.

i.e., $BC^2 = AB^2 + AC^2$.

Arithmetical Note on Pythagoras Theorem

If two sides of a right-angled triangle are given, it is easy to find the third.

Example 1. If two sides of a right-angled triangle are 5 *cm* and 12 *cm*, find the hypotenuse.

$$(\text{Hypotenuse})^2 = h^2 = 5^2 + 12^2 = 169.$$

$$h = \sqrt{169} = 13 \text{ cm}.$$

Example 2. If the hypotenuse and one side of a rt. ∠d △ are 10 *cm* and 6 *cm* respectively, find the other side.

Let x be the other side, we have $10^2 = 6^2 + x^2$ or $x^2 = 100 - 36 = 64$,

$\therefore\ x = \sqrt{64} = 8$ *cm*.

Example 3. The side of a square is a, find its diagonal.

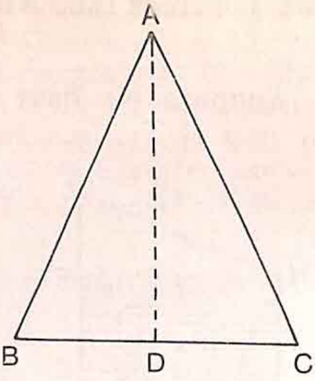
We have $(\text{diagonal})^2 = a^2 + a^2 = 2a^2$.

$\therefore\ \text{diagonal} = \sqrt{2a^2} = a\sqrt{2}$.

That is, *the diagonal of any square is found by multiplying the side by* $\sqrt{2}$.

Example 4. Find the height of an equilateral triangle whose side is a.

We know that the perp. AD from A on BC bisects BC, D being the middle point.



$$\therefore\ AD^2 = AB^2 - BD^2 \text{ (from } \triangle ABD)$$

$$= a^2 - \left(\frac{a}{2}\right)^2 = \frac{3a^2}{4}$$

$$\therefore\ AD = \frac{\sqrt{3}}{2}\,a.$$

That is, *the height of an equilateral triangle is found by multiplying the side by* $\frac{\sqrt{3}}{2}$ or *by* ·866 *nearly.*

Example 5. Find the area of an equilateral triangle whose side is a.

Here base is a.

$\therefore\ \text{height} = \frac{\sqrt{3}}{2}a.$ (Ex. 4 above)

$\therefore\ \text{area} = \frac{1}{2}\text{ base} \times \text{height}$

$= \frac{1}{2}a \times \frac{\sqrt{3}}{2} \times a = a^2 \times \frac{\sqrt{3}}{4}.$

That is, *the area of an equilateral triangle is found by multiplying the square of the sides by* $\frac{\sqrt{3}}{4}$ or by ·433 *nearly.*

Exercise 14

1. Find the hypotenuse of a right-angled triangle when the sides containing the right-angle are

(*i*) 15 *cm*, 36 *cm*, (*ii*) 9 *cm*, 40 *cm*, (*iii*) 96 *cm*, 28 *cm*.

2. Find the remaining side of the following right-angled triangles, given the hypotenuse and one side :

(*i*) hyp. 17 *cm*, side 15 *cm*.

(*ii*) hyp. 50 *cm*, side 48 *cm*.

3. The side of a square is 100 *cm*, find its diagonal.

4. Show that the square on the diagonal of a given square is double of the given square.

5. In $\triangle ABC$, $\angle A = 90°$, $BC = 15$ *cm*, $AB = 9$ *cm*, find AC.

6. The diagonal of a rectangle is 65 *cm* and its breadth is 39 *cm*, find its length.

7. Each side of an isosceles triangle is 26 metres and the base is 20 metres ; find the height.

8. Find the altitude of an equilateral triangle if (*i*) side is 10 *cm*, (*ii*) its perimeter is 48 metres.

9. The diagonal of a square is 20 *cm*, find its area.

10. Determine which of the triangles whose sides are given below are right-angled :

(*i*) 7 *cm*, 24 *cm*, 25 *cm*.

(*ii*) 13, 14, 15.

(*iii*) 8 *cm*, 15 *cm*, 17 *cm*.

(*iv*) 14, 48, 50.

(*v*) $a = 20$ *cm*, $b = 99$ *cm*, $c = 101$ *cm*.

(*vi*) 40, 10, 41.

11. A ladder whose foot is 2·5 metres from the base of a house reaches a window 6 metres above the ground. What is the length of the ladder ?

12. A ladder 10 metres long is so placed that it reaches a ventilator 8 metres above the ground ; and on turning the ladder over to the other side of the lane, its top reaches a window 6 metres above the ground. Find the breadth of the lane.

13. The diagonals of a rhombus are (*i*) 5 *cm*, and 12 *cm* ; (*ii*) 12 *cm* and 16 *cm*. Find their sides.

14. A man travels 27 *km* due South ; then 24 *km* due West ; finally 20 *km* due North. How far is he from the starting point ?

15. From O you go West 25 *km*, then North 60 *km*, then East 80 *km* and finally South 12 *km*. How far are you then from O ?

CHAPTER IV

SOME CONSTRUCTIONS

The students have already done the following constructions in the previous classes :

1. To bisect a given angle 2. To bisect a given straight line. 3. To draw a straight line perpendicular to a given straight line (*i*) at a given point in it, (*ii*) from a given point outside it. 4. At a given point in a given straight line to make an angle equal to a given angle. 5. Through a given point to draw a straight line parallel to a given straight line.

These should be revised before doing the following constructions.

PROBLEM 1

To divide a given finite straight line into any number of equal parts.

Given :—A st. line AB.

Required :—To divide AB into any number of equal parts (*say, seven*).

Steps of Const. :—

1. Draw a st. line AC making a convenient angle with AB.
2. Along AC mark off AP=PQ=QR=RS=ST=TU=UV.
3. Join VB.
4. Through the points P, Q,....U, draw st. lines parallel to VB meeting AB resp. in *p, q, r, s, t, u*.

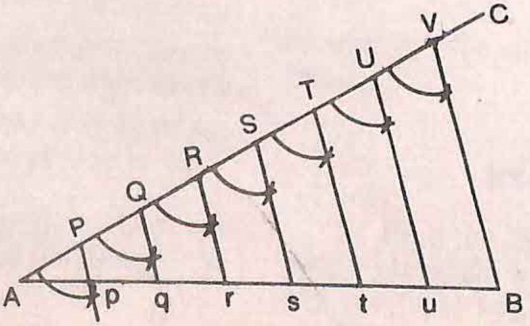
Then the st. line AB is divided into 7 equal parts at *p, q, r, s, t, u*.

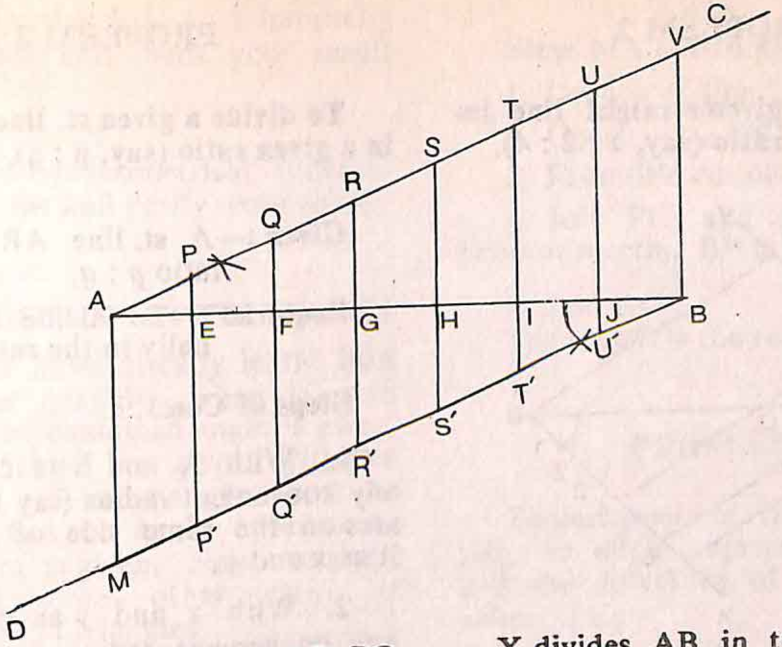
Alternative Method.

Steps of Const :—

1. Draw AC making any convenient angle with AB.

2. Draw BD ∥ CA on the other side of AB.





3. Mark off 7 equal parts AP, PQ RS, ST, TU, UV along AC.

4. Along BD mark off BU′, U′T′, T′S′, S′R′, R′Q′, Q′P′, P′M, each equal to the parts marked along AC.

5. Join AM, PP′, QQ′, RR′, SS′ TT′, UU′, VB. Let PP′, QQ′, RR′, SS′ TT′, UU′ cut AB in E, F, G, H, I, J respectively.

Then AB is divided at E, F, G, H, I, J into 7 equal parts.

Draw a straight line AB 4 *cm* long and take a point X in it such that AX = 3 *cm*.

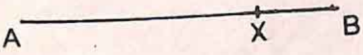


Fig. 1

Then XB = 1 *cm*, and $\frac{AX}{XB} = \frac{3\ cm}{1\ cm} = 3$, *i.e.*, X divides AB in two parts such that one part is three times the other. In other words, we say that X divides the line AB in the ratio of 3 : 1.

Draw another straight line AB = 4 *cm* long and produce it to X so that AX = 6 *cm*. Then BX = 2 *cm* and $\frac{AX}{XB} = \frac{6\ cm}{2\ cm} = 3$ *i.e.*, X divides AB in the ratio of 3 : 1.

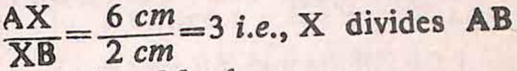
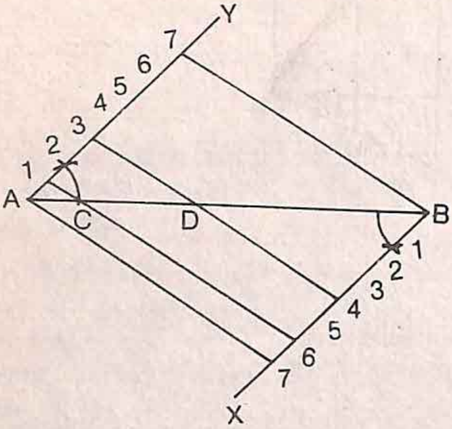


Fig. 2

When the point X lies within A, the point X is said to divide AB internally in the ratio of AX : XB as in Fig. 1. When X lies on AB produced it is said to divide the line AB externally in the ratio of AX : XB as in Fig 2.

PROBLEM 2

To divide a given straight line internally in any ratio (say, 1 : 2 : 4).



Given :—A st. line AB and the ratio 1 : 2 : 4.

Required :—To divide AB internally in the ratio 1 : 2 : 4.

Steps of Const. :

1. Draw AY making any convenient angle with AB.

2. Draw $BX \parallel AY$.

3. Mark off $(1+2+4)=7$ equal parts on AY, as well as on BX marking them 1, 2, 3, 4, 5, 6, 7 each.

4. Join (1, 6) and (3, 4) cutting AB in C and D. Then AC : CD : DB : : 1 : 2 : 4.

PROBLEM 3

To divide a given st. line externally in a given ratio (say, p : q).

Given :—A st. line AB and the ratio $p : q$.

Required :—To divide AB externally in the ratio $p : q$.

Steps of Const. :

1. With A and B as centres and any convenient radius (say Ax), draw arcs on the same side of AB to cut it at x and y.

2. With x and y as centres and any convenient radius draw arcs to cut the former arcs in x' and y'.

3. Join Ax' and By' and cut off $AP = p$ and $BQ = q$.

4. Join PQ and produce it to meet AB *produced* in R.

Then $AR : BR : : p : q$.

Exercise 15

1. Draw a straight line 5·8 *cm* long and by geometrical construction divide it into 6 equal parts.

2. Divide a st. line 7·7 *cm* long internally in the ratio 3 : 8.

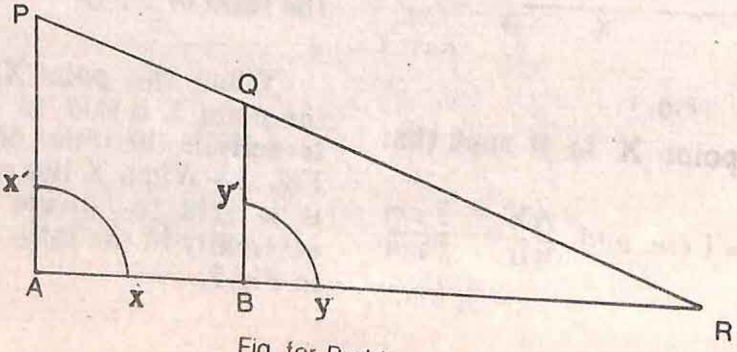


Fig. for Problem 3

3. Take a st. line 6 *cm* long and divide it in the ratio 4 : 5 internally and externally and check your result by calculation.

4. Find by geometrical construction $\frac{3}{7}$ of 7 *cm* and verify your result.

CONSTRUCTION OF TRIANGLES

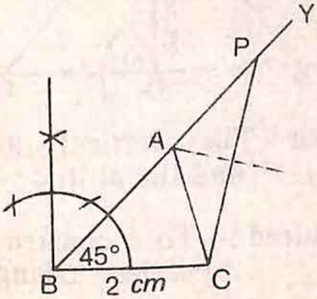
1. You have already learnt how to construct triangles when (*i*) two sides and the contained angle is given, (*ii*) three sides are given, (*iii*) two angles and one side is given and (*iv*) two sides and the angle opposite to one of them is given. Now we shall deal with some other cases of construction of triangles.

It is advisable to draw a rough sketch of a triangle and then mark the data given on it. This helps in suggesting a method for the construction of the triangle.

PROBLEM 4

To construct a triangle when a side, an angle adjacent to the given side and the sum of the other two sides is given.

Given :— BC$=2$ *cm*, $\angle$B$=45°$ and AB$+$AC$=3{\cdot}8$ *cm*.



Required :— To construct the $\triangle$ABC.

Steps of Construction :—

1. Draw a st. line BC$=2$ *cm.*

2. Construct $\angle$CBY$=45°$

3. From BY cut off BP$=3{\cdot}8$ *cm.*

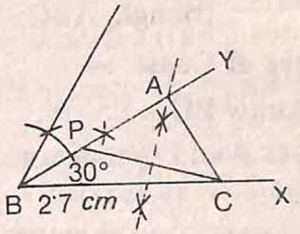
4. Join PC and draw its right bisector meeting BP in A.

5. Join AC.

Then ABC is the required triangle.

PROBLEM 5

To construct a triangle given a side, an angle adjacent to the side and the difference of the other two sides.



Given :— BC$=2{\cdot}7$ *cm*, $\angle$B$=30°$ and AB$-$AC$=1$ *cm.*

Required : —To construct the triangle ABC.

Steps of Const :—

1. Construct $\angle$XBY$=30°$.

2. From BX cut off BC$=2{\cdot}7$ *cm.*

3. From BY cut off BP$=1$ *cm.*

4. Join PC and draw its right bisector meeting BY in A.

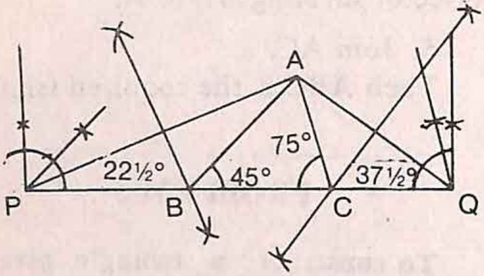
5. Join AC.

Then ABC is the required triangle.

PROBLEM 6

To construct a triangle given its perimeter and base angles.

Given :—Perimeter = 5·5 *cm* and base angles B = 45°, C = 75°



PERIMETER = P Q

Required :—To construct the triangle ABC.

Steps of Const. :—

1. Draw PQ = 5·5 *cm*.

2. At P and Q make angles equal to 45° and 75° respectively.

3. Draw the bisectors of these angles meeting at A.

4. Draw the rt. bisectors of AP and AQ meeting PQ in B and C respectively.

5. Join AB, AC.

Then ABC is the required triangle.

PROBLEM 7

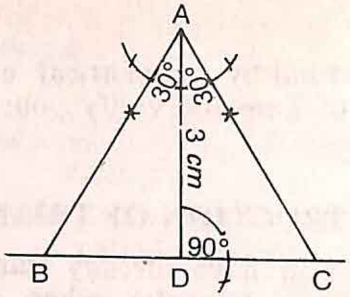
To construct an equilateral triangle having a given altitude.

Given :—The altitude = 3 *cm*.

Required :—To construct an equilateral triangle having this altitude.

Steps of Const. :

1. Draw AD = 3 *cm*.



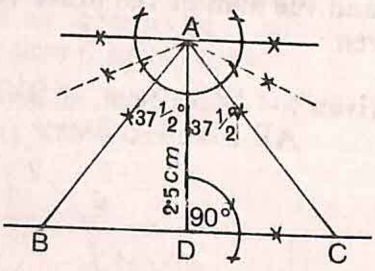
2. At D draw DC⊥AD and produce DC both ways.

3. At A draw straight lines AB and AC making angles of 30° on both sides of AD meeting DC produced in C and B.

Then ABC is the required equilateral triangle.

PROBLEM 8

To construct an isosceles triangle given its vertical angle and the altitude.



Given : The vertical angle = 75° and the altitude = 2·5 *cm*.

Required :—To construct an isosceles triangle with this data.

Steps of Const. :—

1. Draw AD=2·5 *cm*.

2. At D draw $DC \perp AD$ and produce it both ways.

3. At A draw st. lines AB and AC making angles of $37\frac{1}{2}°$ (=half of the verticle angle) on both sides of AD meeting DC produced in B and C.

Then ABC is the required triangle

Exercise 16

1. Construct a triangle ABC such that BC=3·5 *cm*, $\angle B=45°$ and AB+AC=7·2 *cm*. Measure the sides AB, AC and see if their sum equals 7·2 *cm*.

2. Construct a triangle PQR such that QR=4·1 *cm*, $\angle R=30°$ and PQ+PR=8 *cm*.

3. Construct a triangle ABC such that BC=4·8 *cm*., the difference of the other two sides=1·4 *cm* and $\angle B=75°$.

4. Construct a triangle such that its base=3·5 *cm*., one of the angles at the base=45° and the difference of the other two sides=·9 *cm*.

5. Construct a $\triangle ABC$ from the following data :

(*i*) Perimeter=11 *cm*., $\angle B=45°$ and $\angle C=120°$.

(*ii*) Perimeter=7·2 *cm*., $\angle B=60°$ and $\angle A=75°$.

6. Construct an isos. triangle whose perimeter is 8 *cm* and whose vertical angle is of 30°.

7. The perimeter of a right-angled triangle is 10·8 *cm* and one of its angles is 60°. Construct the triangle.

8. Construct equilateral triangles with the altitudes (*i*) 4·5 *cm*. (*ii*) 6 *cm*.

9. Construct an isosceles triangle having an altitude of 3·5 *cm* and the vertical angle=40°. (Use protractor).

10. Construct an isosceles triangle having given the altitude=3·1 *cm* and the vertical angle=75°.

CHAPTER V

CONSTRUCTION OF QUADRILATERALS

1. The straight lines joining the opposite vertices of a quadrilateral are called its **diagonals**.

In the earlier class the student had occasion to draw some quadrilaterals (*i*) when all the four sides and one angle were known, or (*ii*) when three sides and two included angles were known, (*iii*) when four sides and one diagonal were known. The angles could be drawn with the help of the protractor or with compasses.

Here we shall take some examples for practice.

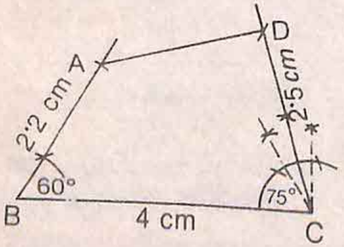
Construction of Quadrilaterals

PROBLEM 9

(*a*) Construct a quadrilateral having given BC=4 *cm*, AB=2·2 *cm*, CD=2·5 *cm* and ∠B=60°, ∠C=75°. Measure AD.

Steps of Const.

1. Draw BC=4 *cm*.



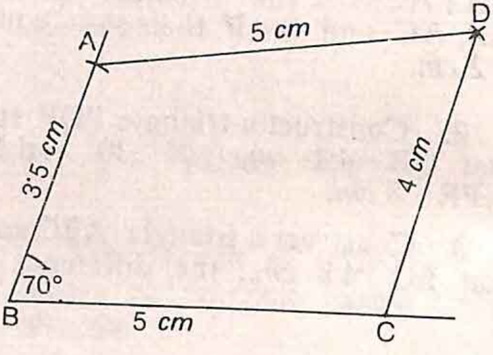
2. At B and C make angles equal to 60° and 75° respectively.

3. Cut off BA=2·2 *cm*, CD=2·5 *cm*.

4. **Join AD.**

Then ABCD is the required quadrilateral. AD=2·5 *cm*.

(*b*) Draw a quadrilateral ABCD given AB=3·5 *cm*, BC=5 *cm*, CD= *cm*, AD=5 *cm*, ∠ABC=70°. Measure ∠BAD.



Steps of Const. :—

1 Draw ∠ABC=70°.

2. Cut off BC=5 *cm*, and BA=3·5 *cm*.

3. With centre A and radius 5 *cm* draw an arc.

4. With centre C and radius 4 *cm*, draw another arc to cut the first arc in D. Join AD, CD.

ABCD is the required quadrilateral.

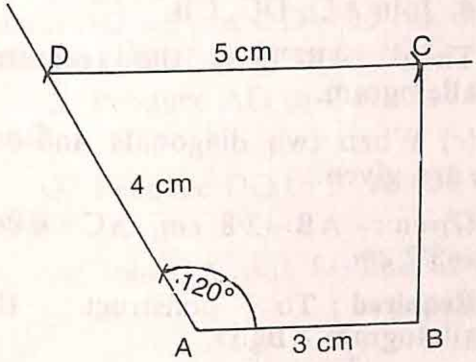
∠BAD=113° (By measurement).

(*c*) Carry out the construction of (*b*) if, instead of $\angle B$, the diagonal $AC = 5{\cdot}2$ *cm* be given.

[**Hint.** First you will construct the $\triangle ABC$, and on AC construct $\triangle DAC$.]

(*d*) Construct the quadrilateral ABCD with the following data :—

$AB = 3$ *cm*, $\angle A = 120^\circ$, $AD = 4$ *cm*.
AB parallel to DC and $DC = 5$ *cm*.



Steps of Const. :—

1. Draw $AB = 3$ *cm*.
2. Make $\angle BAD = 120^\circ$.
3. Cut off $AD = 4$ *cm*.
4. Through D draw DC parallel to AB (by set square or by geometrical construction).
5. Cut off $DC = 5$ *cm*.
6. Join BC.

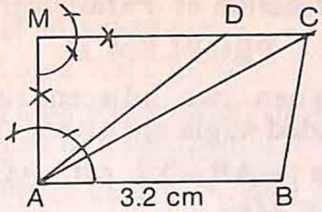
ABCD is the required quadrilateral.

Note. Measure $\angle B$ and see that it is 90°.

Construction of Trapeziums

PROBLEM 10

Construct a trapezium ABCD in which $AB = 3{\cdot}2$ *cm*, $DC = 1$ *cm*, AB parallel to DC, at a distance 2 *cm*, diagonal $AC = 4$ *cm*.



Steps of Const :—

1. Draw $AB = 3{\cdot}2$ *cm*.
2. Draw AM perpendicular to AB and cut off $AM = 2$ *cm*.
3. Through M draw MN parallel to AB.
4. With centre A and radius $= 4$ *cm*, draw an arc cutting MN in C.
5. Cut off $DC = 1$ *cm*. from NM and join AD, BC.

Then ABCD is the required trapezium.

Exercise 17

1. Draw the quadrilateral PQRS, given $\angle P = 65^\circ$, $PQ = 8$ *cm*, $QR = 5$ *cm*, RS 6 *cm*, and $PS = 7$ *cm*.

2. Construct a quadrilateral ABCD, given $AB = 2{\cdot}7$ *cm*, $BC = 2{\cdot}2$ *cm*, $CD = 1{\cdot}5$ *cm*, $DA = 1{\cdot}8$ *cm*, $BD = 3$ *cm*.

3. In a quadrilateral, ABCD, $AB = 4$ *cm*, BC 5 *cm*, $CD = 3$ *cm*, $DA = 6$ *cm*.

Show that the shape of the figure is not fixed by the above data.

Draw the quadrilateral from the above data, when (*i*) $\angle A = 60^\circ$, (*ii*) $\angle A = 90^\circ$

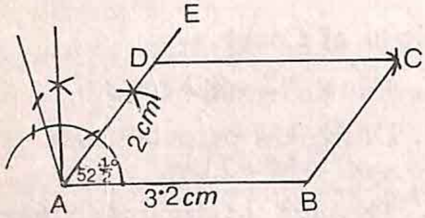
Construction of Parallelograms

PROBLEM 11

(*a*) When two adjacent sides and the included angle are given.

Given :—$AB = 3{\cdot}2$ *cm*, $AD = 2$ *cm* and $\angle A = 52\frac{1}{2}°$.

Required :—To construct the ∥ gm. ABCD.



Steps of Const .

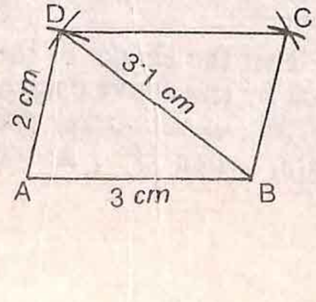
1. Draw $AB = 3{\cdot}2$ *cm.*
2. At A construct $\angle BAE$ $52\frac{1}{2}°$.
3. From AE cut off $AD = 2$ *cm.*
4. With centres B and D and radii equal to 2 *cm* and 3·2 *cm*, respectively, draw two arcs intersecting each other at C.
5. Join BC and CD.

Then ABCD is the required parallelogram.

(*b*) When two adjacent sides and a diagonal are given.

Given :— $AB = 3$ *cm*, $AD = 2$ *cm*, $BD = 3{\cdot}1$ *cm.*

Required :—To construct the parallelogram ABCD.



Steps of Const. :—

1. Draw $AB = 3$ *cm.*
2. With centre A and B and radii equal to 2 *cm* and 3·1 *cm*, respectively draw arcs intersecting each other at D.
3. With centres B and D and radii equal to 2 *cm* and 3 *cm*, respectively, draw arcs intersecting each other in C.
4. Join AD, DC, CB.

Then ABCD is the required parallelogram.

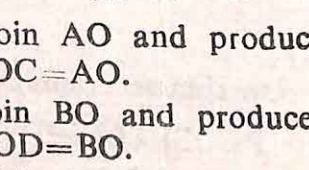
(*c*) When two diagonals and one side are given.

Given : - $AB = 2{\cdot}8$ *cm*, $AC = 4$ *cm*, $BD = 3{\cdot}2$ *cm.*

Required : To construct the parallelogram ABCD.

Steps of Const. :—

1. Draw $AB = 2{\cdot}8$ *cm.*
2. With centres A and B and radii equal to 3 *cm* and 1·6 *cm.* (*equal to half the length of the given diagonals*) describe arcs intersecting each other in O.



3. Join AO and produce it to C so that $OC = AO$.
4. Join BO and produce it to D so that $OD = BO$.
5. Join AD, BC and CD.

Then ABCD is the required parallelogram.

(*d*) When two diagonals and the angle between them are given.

Given :– Diagonals AC=4 *cm*, diagonal BD=3 *cm*, and the angle between them=45°.

Required : To construct the parallelogram ABCD.

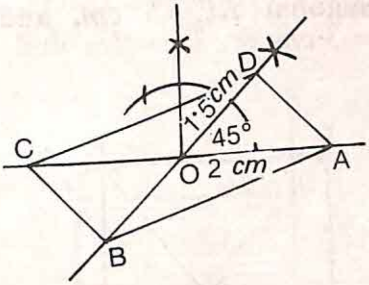
Steps of Const. :

1. Draw ∠AOD=45° with its arms AO=2 *cm*, OD=1·5 *cm*. (*halves the lengths of the diagonals*).

2. Produce AO to C so that OC=AO.

3. Produce DO to B so that OB=OD.

4. Join DA, AB, BC and CD.

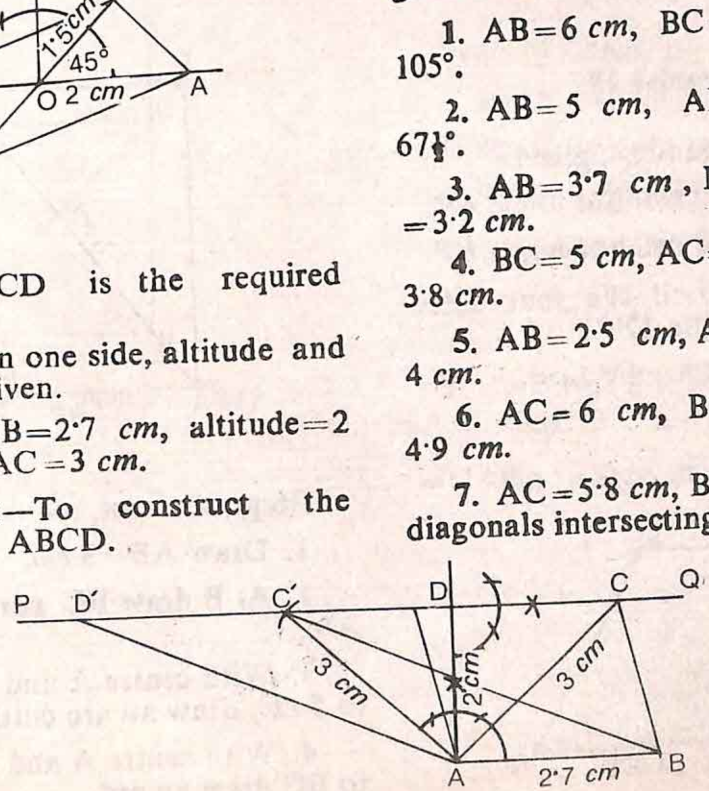


Then ABCD is the required parallelogram.

(*e*) When one side, altitude and diagonal are given.

Given :—AB=2·7 *cm*, altitude=2 *cm*, diagonal AC=3 *cm*.

Required :—To construct the parallelogram ABCD.



Steps of Const :–

1. Draw AB=2·7 *cm*.

2. Draw PQ parallel to AB and at a distance of 2 *cm* from it.

3. With centre A and radius=3 *cm* draw arcs cutting PQ in C and C′.

4. Along QP mark off CD=BA=2·7 *cm*.

5. Again along QP mark off C′D′=BA=2·7 *cm*.

6. Join AD, BC, AD′, BC′.

Then ABCD and ABC′D′ are the *two* required parallelograms.

Exercise 18

Construct a parallelogram ABCD given that

1. AB=6 *cm*, BC=3·5 *cm*, ∠B=105°.

2. AB=5 *cm*, AD=6 *cm*, ∠A=67½°.

3. AB=3·7 *cm*, BC=2·9 *cm*, BD=3·2 *cm*.

4. BC=5 *cm*, AC=4·7 *cm*, AB=3·8 *cm*.

5. AB=2·5 *cm*, AC=3 *cm*, BD=4 *cm*.

6. AC=6 *cm*, BC=6·7 *cm*, AB=4·9 *cm*.

7. AC=5·8 *cm*, BD=7·6 *cm*, these diagonals intersecting at 60°.

8. AB=4·8 *cm*, ∠A=75°, and perpendicular distance between AB and CD=3·2 *cm*.

9. With sides 6·4 *cm* and 4 *cm*, construct a parallelogram which has a height of 3 *cm*.

10. Describe a parallelogram having sides 3·8 *cm* and 6·3 *cm* in length and enclosing an angle of 35°. Measure accurately the lengths of the two diagonals. (Use protractor).

Construction of Rhombus

3. We know that a **rhombus** is a parallelogram which has all its sides equal but its angles are not right angles.

Its construction is the same as given for the parallelograms.

Exercise 19

Construct a rhombus, given

1. One side 4·9 *cm*, one angle 45°.

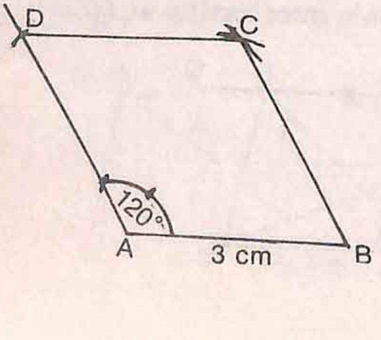
2. One side 5·2 *cm*, one angle 60°.

3. The sum of all the four sides 12 *cm* and one angle 120°

One side = 12 *cm* ÷ 4 = 3 *cm*.

Steps of Const. :—

Draw AB = 3 *cm*. Make ∠BAD = 120° and cut off AD = 3 *cm*. With centres B and D and radii equal to 3 *cm* draw arcs cutting each other at C. Join BC, DC.



Then ABCD is the required rhombus.

Construction of Rectangles

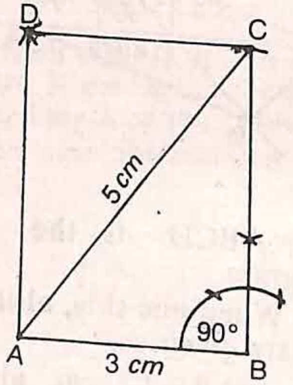
4. **A rectangle is a parallelogram which has one of its angles a right angle.**

Exercise 20

Construct a rectangle ABCD, given

1. AB = 5 *cm*, AD = 2·4 *cm*.

2. Diagonal AC = 5 *cm*, and one side AB = 3 *cm*.



Steps of Const. :—

1. Draw AB = 3 *cm*.

2. At B draw BC perpendicular to AB.

3. With centre A and radius equal to 5 *cm*, draw an arc cutting BC in C.

4. With centre A and radius equal to BC draw an arc.

5. With centre C and radius equal to 3 *cm* draw another arc to cut the first arc in D.

6. Join AD and CD.

Then ABCD is the required rectangle.

3. Draw a rectangle ABCD whose side AB=3·4 *cm* and diagonal AC=5·8 *cm*. Measure the other side.

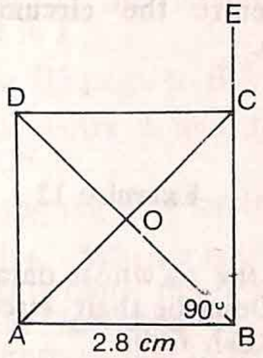
Construction of Squares

5. A rectangle which has all its sides equal is called a square.

Exercise 21

Construct a square ABCD, given.

1. Side AB=2·8 *cm*.



Measure its diagonals. They are equal.

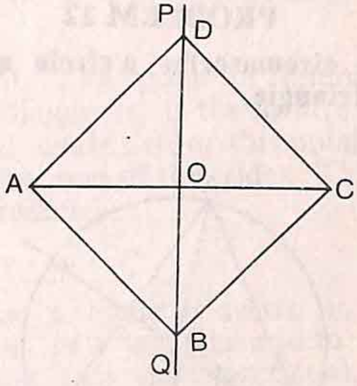
If they intersect in O, AO=OC=BO=DO.

Measure the angles between the diagonals. They are right angles.

2. Perimeter (sum of the four sides) =16 *cm*.

[Here one side=16÷4=4 *cm*.]

3. Diagonal AC=4 *cm*.



Steps of Const. :—

1. Draw AC=4 *cm*.

2. Draw its right bisector POQ.

3. Step off OB=OD=OA.

4. Join AB, BC, CD, DA.

Then ABCD is the required square.

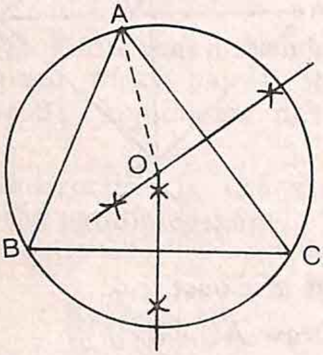
4. Construct a square if its diagonal is 5 *cm* long.

CHAPTER VI

PROBLEMS ON CIRCUMSCRIBING, INSCRIBING AND ESCRIBING CIRCLES

PROBLEM 12

To circumscribe a circle about a given triangle.



Given :—A △ABC, where AB= 3·3 *cm*, BC = 3·5 *cm*, CA = 3·8 *cm*.

Required :—To circumscribe a circle about the △ABC.

Steps of Const. :

1. Draw the rt. bisector of BC.

2. Draw the rt. bisector of AC meeting that of BC in O.

3. With centre O and radius = OA describe a circle.

This is the required circle.

Note. By drawing the figures, it will be found that if the given triangle is *acute-angled*, the centre of the circum-circle falls, *within* it ; if it is a *right-angled* triangle, the centre falls *on the hypotenuse* ; if it is an *obtuse-angled* triangle, the centre falls *outside* the triangle.

Definition. When a circle passes through all the vertices of a figure (here △), it is said to be circumscribed about that figure, and the figure is said to be inscribed in it. This circle is called the **circum-circle** of the figure, its radius the **circum-radius** and its centre the **circum-centre** of the figure.

Exercise 22

Draw the △s whose data are given below. Describe their circum-circles. Measure their radii.

1. AB=2·5 *cm*, BC=3·5 *cm*, AC= 3 *cm*.

2. AB=5·2 *cm*, BC=5·6 *cm*, CA= 6 *cm*.

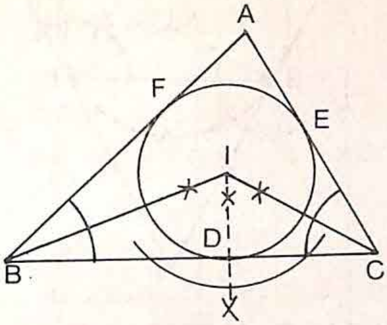
3. a=2·5 *cm*, b=6 *cm*, c=6·5 *cm*.

4. c=4 *cm*, a=5 *cm*, ∠B=120.

5. AB=3·8 *cm*, BC=4 *cm*, CA= 5 *cm*.

PROBLEM 13

To inscribe a circle in a given triangle.



Given :—A triangle ABC where BC=5 *cm*, CA=3·5 *cm*, and AB=4·5 *cm*.

Required :—To inscribe a circle in the △ABC.

Steps of Const. :—

1. Draw BI, the bisector of ∠B.

2. Draw CI, the bisector of ∠C, meeting BI in I.

3. Draw ID perp. to BC.

4 With centre I and radius ID, draw a circle.

This is the required circle.

Definition. When a circle touches all the sides of any figure (here △) it is said to be **inscribed** in the figure and the figure is said to be **described** about the circle. This circle is called the **incircle** of the figure, its radius the **inradius** and its centre, the **incentre** of the figure.

Exercise 23

1. Inscribe a circle in a triangle whose sides are :

(*a*) 6 *cm*, 8 *cm*, 10 *cm*. (*b*) 5·6 *cm*, 5·2 *cm*, 4·8 *cm*.

Measure the radius in each case.

2. Construct a △ whose sides are 5·2 *cm*, 5·6 *cm* and 6 *cm*. Inscribe a circle in it and measure its radius.

3. Construct a △ of sides 6, 7 and 8 *cm* respectively ; draw the inscribed circle of this △ and measure the radius.

4. Inscribe a circle (*a*) in a given square, (*b*) in a given rhombus.

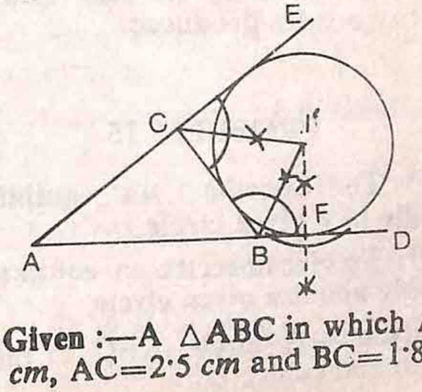
[**Hint.** The point of intersection of the diagonals is the centre of the required circle ; from this point draw a **perp.** to one of the sides. This will be the radius.]

When a circle is drawn to touch one side of a triangle and to touch the other two sides produced, then that circle is called an **escribed circle** or an **ex-circle** of the triangle. Its radius is called the **ex-radius** and its centre the **ex-centre** of the triangle.

Clearly, there will be three ex-circles of a triangle.

PROBLEM 14

To escribe a circle to a given triangle.



Given :—A △ABC in which AB=3·7 *cm*, AC=2·5 *cm* and BC=1·8 *cm*.

Required :—To draw the escribed circle of the △ABC touching BC and the sides AB, AC produced to D and E, respectively.

Steps of Construction :—

1. Draw BI′ and CI′ the bisectors of the ∠s DBC, ECB meeting at I′.

2. Draw I′F perp. to AD.

3. With I′ as centre and radius= I′F draw a circle.

This is the required circle.

Note. It is clear that every triangle has *three* escribed circles. The radii of the escribed circles touching the sides BC, CA, AB of the △ABC, respectively, are usually denoted by r_1, r_2, r_3.

Exercise 24

1. Draw a △ with sides 6 *cm*, 5·6 *cm*, 5·2 *cm*. Escribe a circle touching the shortest side. Measure the radius.

2. Draw the three escribed circles of the △ whose sides are 3, 3·5 and 5 *cm*. Measure the radii.

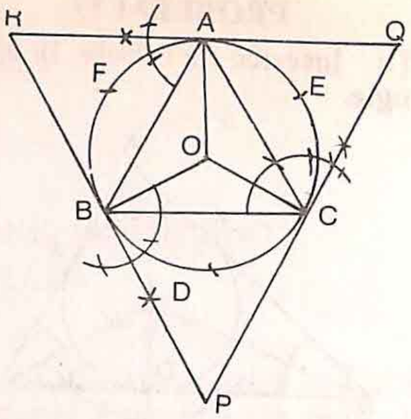
3. Draw a △ whose sides are 2·4 *cm*, 3·6 *cm*, 4·8 *cm*. Inscribe a circle in it and also describe a circle touching the smallest side and the other two sides produced.

PROBLEM 15

(*i*) To inscribe an equilateral triangle in a given circle.

(*ii*) To circumscribe an equilateral triangle about a given circle.

Given :—A circle ABC of radius 1·5 *cm* and centre O.



Required : (*i*) To inscribe an equilateral △ in the circle ABC.

(*ii*) To circumscribe an equilateral △ about the circle ABC.

(*i*) **Steps of Construction** :—

1. Along the O^{ce}, step off 6 equal parts each equal to the radius of the circle, *i.e.*, 1·5 *cm*

2. Join A, B, C, the *alternate* points of division. Then ABC is the required triangle.

(*ii*) **Steps of Construction** :—

1. Same as in (*i*) above.

2. At A, B, C draw perps., to OA, OB, OC, resp., meeting and forming the △ PQR.

This is the required triangle.

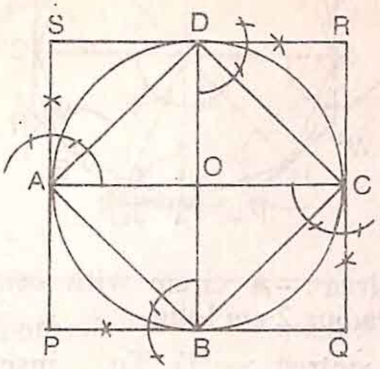
Exercise 25

Inscribe an equilateral △ in a circle whose diameter is 4·8 *cm*. Also circumscribe an equilateral △ about the circle. Measure the sides

PROBLEM 16

(*i*) To inscribe a square in a given circle.

(*ii*) **To circumscribe a square about a given circle.**



Given :—A circle with centre O and radius = 1·9 *cm*.

Required :—(*i*) To inscribe a square in this circle.

(*ii*) To circumscribe a square about this circle.

(*i*) **Steps of Construction** :—

1. Draw any diameter AC.

2. Draw another diameter BD ⊥ AC.

3. Join AB, BC, CD, DA.

Then ABCD is the required square.

(*ii*) **Steps of Construction** :—

1 and 2 same as in (*i*) above.

3. Draw perps. at A, B, C and D forming the figure PQRS.

Then PQRS is the required square.

Exercise 26

1. Inscribe a square in a circle of 2·5 *cm* radius. Measure and calculate its side.

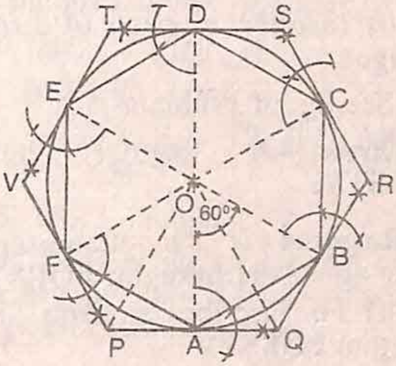
2. About a circle of 4·5 *cm* radius describe a square.

3. Prove that the area of a square circumscribed about a given circle is double the area of an inscribed square.

PROBLEM 17

(*i*) **To inscribe a regular hexagon in a given circle.**

(*ii*) **To circumscribe a regular hexagon about a given circle.**



Given :—A circle with a centre O and radius 2 *cm* long.

Required : (*i*) To inscribe a regular hexagon in it.

(*ii*) To circumscribe a regular hexagon about it.

(*i*) **Steps of Const.** :—

1. Draw any radius OA and starting from A step off six equal parts, each equal to OA along the circumference.

2. Join the pts. of division A, B, C, D, E, F.

Then ABCDEF is the required inscribed hexagon.

(*ii*) **Steps of Construction** :—

1. As in (*i*) above.

2. At the pts. A, B, C, D, E, F

draw perps. to OA, OB, OC, OD, OE, OF resp., to meet and form the figure PQRSTV, which is the required hexagon.

Exercise 27

1. Inscribe a regular hexagon in a circle of 1·5 *cm* radius and circumscribe a regular hexagon about a circle of 1·3 *cm* radius. Measure the side in each case.

2. (*i*) *Circumscribe a circle about a regular hexagon.*

(*ii*) *Inscribe a circle in a regular hexagon.*

(See fig. of Problem 17.)

Given :—A regular hexagon PQRSTV.

Required : (*i*) To circumscribe a circle about the hexagon PQRSTV.

(*ii*) To inscribe a circle in the hexagon PQRSTV.

(*i*) **Steps of Construction** :—

1. Draw PO, QO, the bisectors of the angles P, Q intersecting in O.

2. With centre O and radius OP draw a circle, which will pass through Q, R, S, T, V.

(*ii*) **Steps of Construction** :—

1. as in (*i*) above.

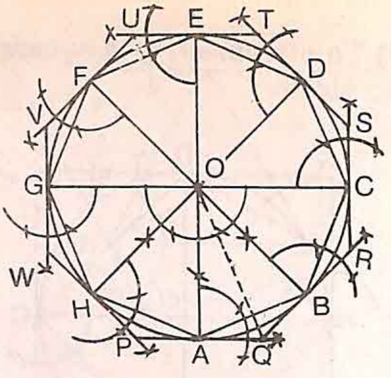
2. Draw OA ⊥ PQ, OB ⊥ QR, etc.

3. With centre O and radius OA draw a circle, which will be the required inscribed circle.

PROBLEM 18

(*i*) To inscribe a regular octagon in a given circle.

(*ii*) To circumscribe a regular octagon about a given circle.



Given :—A circle with centre O and radius 2 *cm* long.

Required :—(*i*) To inscribe a regular octagon in it.

(*ii*) To circumscribe a regular octagon about it.

(*i*) **Steps of Construction** :—

1. Draw any diameter GC.

2. Draw another diameter EA ⊥ GC.

3. Draw the bisectors HOD, FOB of the ∠s at O.

4. Join the pts. of division on the O^ce to get the fig. ABCDEFGH.

This is the required octagon.

(*ii*) **Steps of Construction** :

1 to 3 same as in (*i*) above.

4. At A, B, C, D, E, F, G, H draw perps. respectively, to OA, OB, OC, OD, OE, OF, OG, OH to meet and form the required octagon PQRSTUVW.

Exercise 28

1. In a circle of radius 5·8 *cm* inscribe a regular octagon. Measure its sides.

2. About a circle of 2·2 *cm* radius, circumscribe a regular octagon and measure its side.

ANSWERS

Exercise 1

1.

Hypothesis	*Conclusion*
(*a*) Two straight lines intersect.	Vertically opposite angles are equal.
(*b*) A triangle.	Sum of the three angles is two right angles.
(*c*) Two right-angled triangles in which a side and the hypotenuse of one are respectively equal to the side and hypotenuse of the other triangle.	The triangles are congruent.
(*d*) A triangle in which two sides are equal.	Angles opposite to those sides are equal.
(*e*) A straight line cutting two other straight lines and making a pair of alternate angles equal.	The two straight lines are parallel.

2. No.

3. (*a*) If the sum of two adjacent angles is equal to two right angles, their external arms lie in the same straight line.

(*b*) If two sides of a triangle are equal, the angles opposite to them are equal.

(*c*) If a straight line cuts two parallel straight lines, the corresponding angles are equal.

(*d*) In a triangle, the greater angle has the greater side opposite to it.

4. (*i*) $\angle A$. (*ii*) $\angle Q$. (*iii*) $\angle M$.
(*iv*) $\angle Y$. (*v*) $\angle E$.

5. (*i*) QR. (*ii*) XZ. (*iii*) AB.
(*iv*) LN.

6. BC. 7. AB. 8. $\angle A$.

9. None of the triangles is possible because the sum of any two sides is not greater than the third.

Exercise 2

1. (*i*) 120°, (*ii*) 135°, (*iii*) 144°.

2. 36°, 72°, 108°, 144°. 3. 165°.

6. (*i*) 6, (*ii*) 8, (*iii*) 10.

7. (*i*) Yes, (*ii*) Yes, (*iii*) No,
(*iv*) No, (*v*) Yes.

9. (*i*) 45°, (*ii*) 36°.

10. (*i*) 12, (*ii*) 15.

Exercise 3

1. (*i*) Rhombus, (*ii*) Parallelogram,
(*iii*) Square, (*iv*) Rectangle.

2. (*i*) Opposite, (*ii*) Parallelogram,
(*iii*) Parallel and equal,
(*iv*) Rhombus, (*v*) Diagonal.

3. (*i*) $AB=3$ *cm*. $BC=2$ *cm*, $\angle B=65°$, $\angle C=\angle A=115°$.

(*ii*) $PS=PQ=QR=3$ *cm*, $\angle Q=60°$, $\angle P=\angle R=120°$.

4. (*i*) In a parallelogram, the opposite sides are equal.

(*ii*) A quadrilateral is a parallelogram if its opposite angles are equal.

5. Rectangle 6. Rhombus.

Exercise 12

1. (*a*) 52 sq. *cm* (*b*) 6 *cm*.

Exercise 13

1. (*i*) 2 sq. *cm*, (*ii*) 2·7 sq. *cm*.

2. 8 *cm*. 3. 3 sq. *cm*.

4. (*i*) 7 sq. *cm*. (*ii*) 3 sq. *cm*.

4. 13 sq. *cm*.

Exercise 14

1. (*i*) 39 *cm*, (*ii*) 41 *cm*, (*iii*) 100 *cm*.

2. (*i*) 8 *cm*, (*ii*) 14 *cm*.

3. 141·42 *cm*. app. 5. 12 *cm*.

6. 52 *cm*. 7.24 metres.

8. (*i*) 8·66 *cm*, (*ii*) 13·856 metres.

9. 200 sq. *cm*. 10. (*i*), (*iii*), (*iv*), (*v*) are rt.-angled $\triangle$s ; (*ii*), (*vi*) are not rt.-angled $\triangle$s.

11. 6·5 metres. 12. 14 metres.

13. (*i*) 6·5 *cm*. (*ii*) 10 *cm*.

14. 25 KM. 15. 73 KM

Exercise 22

1. 1·8 *cm*,nearly. 2. 3·25 *cm*.

3. 3·25 *cm*. 4. 4·5 *cm* nearly

5. 2·5 *cm*.

Exercise 23

1. (*a*) 2 *cm*, (*b*) 15 *cm* nearly.

2. 1·6 *cm*, 3. 1·9 *cm* nearly.

Exercise 24

1. 4·2 *cm*. 2. 6·9 *cm* ; 2·3 *cm* ; 1·9 *cm* nearly.

Exercise 25

1. 4·2 *cm* nearly ; 8·3 *cm* nearly.

Exercise 26

1. 3·5 *cm* nearly.

Exercise 27

1. 1·5 *cm*, 1·5 *cm* nearly.

Exercise 28

1. 4·4 *cm*. 2. 1·8 *cm*.